HELSINKI UNIVERSITY OF TECHNOLOGY
Signal Processing Laboratory
S-88.211 Signal Processing in Telecommunications I (2 cu)

Exam 14 May 2004
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Write in each answer paper your name, department, student number, the course name and the date. Number each paper you submit and denote the total no. of pages. The exam paper is only available in English, but please feel free to write in Finnish or Swedish if you prefer. 5 problems, 30 points total. The BETA mathematical tables can be utilized – you can borrow a copy from the exam supervisor if you do not have your own.

1. (1p each) Define and describe *briefly* (2..3 lines of text) the following concepts:
   a) Zero-forcing equaliser
   b) OFDM
   c) Water-pouring theorem
   d) Matched filter
   e) Nyquist criterion
   f) MLSD

2. (6p) Define and explain the matched filter for an AWGN channel . Define it in the frequency domain and derive the time-domain form. Explain how it can be combined with Nyquist's ideas for practical Tx and Rx filter design.

3. Matched filters. Consider a discrete-time receive filter $h_R(k)$ and its frequency response $H_R(e^{j\omega k})$. Assume a simple discrete-time transmit filter:

$$h_T(k) = \delta(k) + 2\delta(k-1) + \delta(k-2) \tag{1}$$

a) (3p) Find the matched-filter receive filter $h_R(k)$ and draw the impulse responses $h_T(k)$ and $h_R(k)$, both ideal noncausal and causal versions.

b) (3p) Determine the pulse waveform $g(k)$ at the output of the receive filter either via convolution:

$$g(k) = h_R(k) * h_T(k) = \sum_{l=-\infty}^{\infty} h_R(l) h_T(k-l) \tag{2}$$

or in the frequency domain if you prefer. Plot $g(k)$.

4. Adaptive filters. Let us consider a discrete-time model for a communication system in a linear channel (sampled at symbol rate). The received signal samples $r(k)$ are filtered by an N-tap FIR filter (equalizer). The equalizer output can be expressed as

$$y(k) = \mathbf{h}_R^T \mathbf{r}(k). \tag{3}$$

where $\mathbf{h}_R$ and $\mathbf{r}$ are N-dimensional column vectors. The mean squared error (MSE) can be expressed as

$$E\left[e^2(k)\right] = E\left[a_k^2\right] - 2\mathbf{h}_R \mathbf{p}_0 + \mathbf{h}_R^T \mathbf{R} \mathbf{h}_R \tag{4}$$

where $\mathbf{p} = E\lfloor \mathbf{r}(k)a_k \rfloor$ and $\mathbf{R} = E\lfloor \mathbf{r}(k)\mathbf{r}^{\mathrm{T}}(k) \rfloor$.

a) (2p) Derive the optimal minimum-MSE equalizer.
b) (2p) The general structure for an adaptive equalizer using *MSE gradient* (MSEG) algorithm is of the form

$$\mathbf{h}_{\mathrm{R}}[j+1] = \mathbf{h}_{\mathrm{R}}[j] - \frac{\beta}{2}\nabla_{h_{\mathrm{R}}} \mathrm{E}\left[e^2(k)\right]. \tag{5}$$

Derive the MSEG algorithm for the equalizer.

c) (2p) In practice, usually we do not know **p** or **R**. However, we can assume that we have access to a reference signal containing the correct symbol sequence a_k (e.g., a known training signal or decided symbols) so that we can compute the error

$$e(k) = y(k) - x(k). \tag{6}$$

Derive the *stochastic gradient* (SG) algorithm for the filter which uses the *instantaneous* squared error to determine the gradient estimate.

5. (9p)
Describe and compare DFE equalizer and Viterbi algorithm (the latter provided with a simple channel estimator). Discuss the advantages and disadvantages applications like in mobile communications channels (with fading) and wireline channels.