HELSINKI UNIVERSITY OF TECHNOLOGY
Signal Processing Laboratory
S-88.211 Signal Processing in Telecommunications I (2 cu)

Exam 16 May 2003
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Write in each answer paper your name, department, student number, the course name and the date. Number each paper you submit and denote the total no. of pages. The exam paper is only available in English, but please feel free to write in Finnish or Swedish if you prefer. 5 problems, 30 points total. The BETA mathematical tables can be utilized – you can borrow a copy from the exam supervisor if you do not have your own.

1. (1p each) Define and describe *briefly* (2..3 lines of text) the following concepts:
   a) MMSE equalizer
   b) Echo canceller
   c) Root-Raised-Cosine (RRC) filter
   d) Crosstalk
   e) ADSL
   f) Viterbi algorithm

2. (6p) Nyquist criterion

3. Matched filters
   a) (3p) Let us consider a discrete-time channel with the impulse response

$$c(k) = 5\delta(k) + 2\delta(k-2) + \delta(k-3) \tag{1}$$

Design a receive filter $h_R(k)$ that is matched to the channel (i.e., a filter that maximizes the received SNR when assuming that the transmit filter is $h_T(k) = \delta(k)$) and is also causal.

b) (3p) In CDMA receivers, a so-called RAKE receiver is used to utilize the diversity of multipath channels. Describe its operation with a block diagram and define the optimal RAKE receiver that collects the signal energy from the channel in the best possible way (also called Maximum Ratio Combiner). Assume that the intersymbol interference can be neglected.

4. Adaptive filters EXTRA

Let us consider a discrete-time model for a communication system in a linear channel (sampled at symbol rate). The received signal samples $r(k)$ are filtered by a one-tap FIR filter (equalizer) with the coefficient $h_R(0) = h_R$. The equalizer output can be expressed as

$$y(k) = h_R r(k)\,. \tag{2}$$

The mean squared error (MSE) can be expressed as

$$E\left[e^2(k)\right] = E\left[a_k^2\right] - 2h_R(0)p_0 + R_0 h_R^2(0) \tag{3}$$

where $p_0 = E\left[r(k)a_k\right]$ and $R_0 = E\left[r^2(k)\right]$.

a) (2p) Derive the optimal minimum-MSE equalizer.

b) (3p) The general structure for an adaptive equalizer using *MSE gradient* (MSEG) algorithm is of the form

$$h_{\mathrm{R}}[j+1] = h_{\mathrm{R}}[j] - \frac{\beta}{2}\nabla_{h_{\mathrm{R}}} \mathrm{E}[e^2(k)]. \tag{5}$$

Derive the MSEG algorithm for the 1-tap equalizer.

c) (4p) In practice, usually we do not know p_0 or R_0. However, we can assume that we have access to a reference signal containing the correct symbol sequence a_k (e.g., a known training signal or decided symbols) so that we can compute the error

$$e(k) = y(k) - x(k) = h_{\mathrm{R}} r(k) - a_k. \tag{6}$$

Derive the *stochastic gradient* (SG) algorithm for the 1-tap filter which uses the *instantaneous* squared error to determine the gradient estimate.

NOTE! By solving all the subproblems a)-c) correctly you can gain 3 extra points!

5. Capacity of mobile communications

In mobile communications, the channel may vary with time, distance etc. Let us assume that the channel response is a real-valued constant, i.e. $C(f) = K$ and the channel noise is AWGN with the power spectrum $S_n(f) = P_n/(2W)$. $W = 4$ kHz and $SNR = P_x/P_n = 15$ dB.

a) (3p) Solve for the optimum transmit power spectrum $S_x(f)$ that maximizes the channel capacity when the total transmit power P_x is limited.

b) (2p) Determine the general expression for the channel capacity and the numerical value in this case (assume $K = 1$).

c) (1p) The channel fluctuations are modeled by assuming that K takes values 0.5, 1, and 2 equally often. Determine the average channel capacity.

Hints: The optimal power spectrum is obtained with the water-pouring theorem as

$$S_{x,opt}(f) = L - S_n(f)/|C(f)|^2 \tag{2}$$

where L is determined so that the total transmit power

$$P_x = \int_{-\infty}^{\infty} S_{x,opt}(f)\,df \tag{3}$$

is limited. The capacity is then obtained by integration

$$C = \int_0^{\infty} \log_2\left(1 + \frac{S_x(f)|C(f)|^2}{S_n(f)}\right) df = \frac{1}{2}\int_{-\infty}^{\infty} \log_2\left(1 + \frac{S_x(f)|C(f)|^2}{S_n(f)}\right) df \tag{6}$$