**AS-74. 3123 Model-Based Control Systems**
**Exam 4. 9. 2008**

The questions are available only in English. You can answer in Finnish, Swedish or English. The final grade is given when both the examination and the homework problem have been accepted.

5 problems.

**1.** The first-order discrete system

$$x(k+1) = 0.5x(k) + u(k), \quad x(0) = x_0$$

is to be transformed to the origin in two steps ($x(2) = 0$) while the performance measure

$$J = \sum_{k=0}^{1} \left[ |x(k)| + 5|u(k)| \right]$$

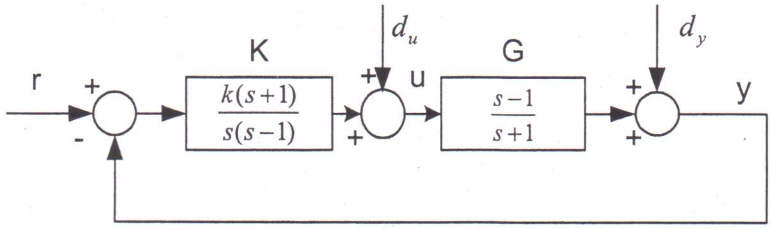
is minimized.

Use the method of dynamic programming to determine the optimal control law $\{u^*(0), u^*(1)\}$, when the initial state of the system is $x(0) = -2$. Assume that the admissible control values are 1, 0.5, 0, -0.5, -1. What is the optimal cost?

**2.** Explain briefly the following concepts

- **a.** singular values
- **b.** $H_\infty$-norm
- **c.** internal stability
- **d.** "Relative gain array" (RGA)
- **e.** LQ-control
- **f.** "Internal model control" (IMC)

**3.** Consider the control configuration in the figure, in which the parameter $k$ is positive.




**a.** Assuming $r = 0$ express the outputs $u(s)$ and $y(s)$ as functions of the inputs $d_u(s)$, $d_y(s)$.
**b.** Calculate and plot (schematically only) the frequency responses of the loop gain and sensitivity functions.
**c.** Are there any pole-zero cancellations? Is the system internally stable?

**4.** Consider a SISO-system in a one-degree-of-freedom control configuration. The connection between the real and nominal system is

$$G_0(s) = G(s)(1+\Delta_G(s))$$

Derive a condition to the system to be robustly stable.

**5.** Consider the system

$$\dot{x} = u, \quad x(0) = 1$$

and the cost function to be minimized

$$J = \frac{1}{2}\int_0^2 \left(x^2 + u^2\right) dt$$

**a.** Determine the optimal control law.
**b.** Calculate $x^*(1)$. What is the total optimal cost?

**Some formulas, which might be useful:**

$$\dot{x} = Ax + Bu, \quad t \ge t_0$$

$$J(t_0) = \frac{1}{2}x^T(T)S(T)x(T) + \frac{1}{2}\int_{t_0}^{T}\left(x^T Q x + u^T R u\right)\ dt$$

$$S(T) \ge 0, \quad Q \ge 0, \quad R > 0$$

$$-\dot{S}(t) = A^T S + SA - SBR^{-1}B^T S + Q,\ t \le T,\ \text{boundary condition } S(T)$$

$$K = R^{-1}B^T S$$

$$u = -Kx$$

$$J^*(t_0) = \frac{1}{2}x^T(t_0)S(t_0)x(t_0)$$

$$\int \frac{dx}{x^2 - 1} = \frac{1}{2}\ln\left|\frac{x-1}{x+1}\right| + C$$