AS-74. 3123 Model-Based Control Systems
Exam 10. 1. 2008

The questions are available only in English. You can answer in Finnish, Swedish or English. The final grade is given when both the examination and the homework problem have been evaluated and accepted.

5 problems.

1. a. Consider a linear multivariable (MIMO) system. Draw a schema of the "two-degrees-of-freedom" control configuration. Define the concepts *closed-loop transfer function*, *sensitivity function* and *complementary sensitivity function* for it.

b. As in problem 1a, draw a schema of the "one-degree-of freedom" control configuration and show how it can be changed into the "two-degrees-of-freedom" form. By using the results in problem 1a, define the *closed-loop transfer function*, *sensitivity function* and *complementary sensitivity function* for this configuration.

c. Derive the following relationships (the variables are standard used during the course)

$$(I+L)^{-1}L = I-(I+L)^{-1}$$
$$T=(I+L^{-1})^{-1}$$

2 a. Explain the *Small Gain Theorem.*

b. Explain shortly what is meant by the Relative Gain Array (RGA) and what is its meaning in control engineering.

c. Consider a SISO-system in one-degree-of-freedom configuration. The relationship between the real and nominal system is

$$G_0(s)=G(s)(1+\Delta_G(s))$$

where the variables represent standard symbols used in the course. Derive the condition for the system to be robustly stable. Explain verbally what robust stability means.

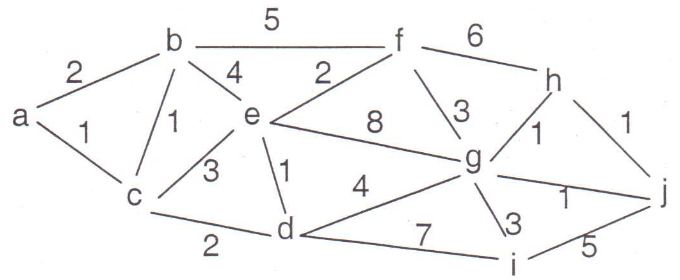
3. a. Consider a linear SISO system. Explain shortly what different definitions there exist for the concept *bandwidth*. Explain these shortly. How can they be characterized in terms of control performance?

b. For the input-output static system matrix

$$G = \begin{bmatrix} 100 & 0 \\ 0 & 1 \end{bmatrix}$$

calculate the eigenvalues and singular values. Explain the relevance of these when considering control performance.

4. In the below figure the cost of moving from one node to another is given by the numbers; movement is allowed only from left to right.



Use dynamic programming to solve the following two problems:

a. Find the minimum cost path from node a to the desired final state j.

b. Now, define the *target set* S as $\{h, i, j\}$. That is, any value of the final state within this set is acceptable. Find the minimum cost path from a to S.

5. For the system

$$\dot{x}(t) = 2u(t)$$

$e^{-t\sqrt{2}}$

($x(0) = x_0$) calculate the control law, which minimizes the criterion

e^{-2t}

$$J = \frac{1}{2}\int_0^{\infty}(x^2(\tau) + u^2(\tau))d\tau$$

What is the optimal cost? What is the closed-loop trajectory of the system? Verify the value of optimal cost by substituting $x^*(t)$ and $u^*(t)$ into the integral formula of J and then calculating its value.

Some formulas, which might be useful:

$\dot{x} = Ax + Bu, \quad t \geq t_0$

$J(t_0) = \frac{1}{2}x^T(T)S(T)x(T) + \frac{1}{2}\int_{t_0}^{T}\left(x^T Qx + u^T Ru\right) \; dt$

$S(T) \geq 0, \quad Q \geq 0, \quad R > 0$

$-\dot{S}(t) = A^T S + SA - SBR^{-1}B^T S + Q,\ t \leq T,$ boundary condition $S(T)$

$K = R^{-1}B^T S$

$u = -Kx$

$J^*(t_0) = \frac{1}{2}x^T(t_0)S(t_0)x(t_0)$