Tfy-99.4280 Medical Imaging Methods

Problems for examination on May 16, 2008

1. Positron Emission Tomography is one of the most efficient imaging tools in current cancer diagnostics. Explain the physical and technical working principle of a PET scanner. What are the inherent weaknesses of this method?

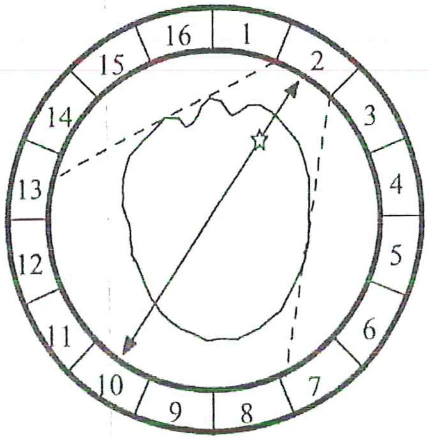
2. In a technetium generator the ^{99m}Tc generated by decay of ^{99}Mo is being "milked" out at 24 h intervals for 7 days. Calculate how much this deviates from the theoretically best interval for reaching the maximum ^{99m}Tc activity after each milking. How does this time depend on the milking history?

3. You would like to try an ultrasonic imaging device for studying a long circular bone inside a muscle. Explain why it might be difficult to obtain images with clinically adequate quality.

4. In MRI the time evolution of x, y and z components of magnetization of the proton system after an RF pulse is described by 3 coupled differential equations. How these equations are called? Describe qualitatively how M_z and M_y develop after a 90° pulse. Explain the physical basis of the decay times T_1 and T_2 as well as their role in enabling to identify various tissues.

5. What is the name of the mathematical operation resulting to a sinogram of function f(x,y). Sketch a sinogram $p(r,\phi)$ for values of ϕ from 0 to 360° for an object made of lead and having a profile with shape of capital letter M. In which imaging method the sinogram concept is utilized and for what purpose?

- **The attached selected lecture material is at your disposal**
- **You may answer in English, Finnish or Swedish**



The on-site technetium generator consists of an alumina ceramic column with radioactive ^{99}Mo absorbed on its surface in the form of ammonium molybdenate. The column is housed within a lead shield for safety considerations. The ^{99m}Tc is obtained by flowing an eluting solution of saline through the generator. The solution washes out the ^{99m}Tc, which binds very weakly to the alumina, leaving the ^{99}Mo behind. Suitable radioassays are then carried out to determine the concentration and the purity of the eluted ^{99m}Tc. Typically, the technetium is eluted every 24 h and the generator is replaced once a week. A simple mathematical model, presented below, describes the dynamic operation of the technetium generator.

The number of ^{99}Mo atoms, denoted by N_1, decreases with time from an initial maximum value N_0 at time $t = 0$. This radioactive decay produces N_2 atoms of ^{99m}Tc, which decay to form N_3 atoms of ^{99}Tc, the final stable product:

$$\underset{(N_1)}{^{99}\mathrm{Mo}} \xrightarrow{\lambda_1} \underset{(N_2)}{^{99m}\mathrm{Tc}} \xrightarrow{\lambda_2} \underset{(N_3)}{^{99}\mathrm{Tc}}$$

The values of λ_1 and λ_2 are 2.92×10^{-6} and 3.21×10^{-5} s^{-1}, respectively. In the following analysis, for simplicity, the time dependence of N_1, N_2, and N_3 is assumed, rather than explicitly stated as $N_1(t)$, etc. The decay process can be represented by three simple differential equations:

$$\frac{dN_1}{dt} = -\lambda_1 N_1, \qquad \frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2, \qquad \frac{dN_3}{dt} = +\lambda_2 N_2 \tag{2.5}$$

Applying the boundary condition that $N_2 = 0$ at $t = 0$, we obtain

$$C = -\frac{\lambda_1 N_0}{\lambda_2 - \lambda_1} \tag{2.12}$$

The final solution for N_2 is therefore

$$N_2 = \frac{\lambda_1 N_0}{\lambda_2 - \lambda_1}(e^{-\lambda_1 t} - e^{-\lambda_2 t}) \tag{2.13}$$

The radioactivity of ^{99m}Tc, Q_2, is thus given by

$$Q_2 = \frac{\lambda_1 \lambda_2 N_0}{\lambda_2 - \lambda_1}(e^{-\lambda_1 t} - e^{-\lambda_2 t}) \tag{2.14}$$

The time dependence of both Q_1 and Q_2 is plotted in the left part of Figure 2.3. The fact that the half-life (66 h) of the "parent" element, ^{99}Mo, is an order of magnitude longer than that (6 h) of the "daughter" isotope, ^{99m}Tc, results in an equilibrium state being established in which the ratio of the amounts of the two species is constant, that is, the decay rate of the daughter nucleus is governed by the half-life of the parent, rather than by its own. In practice, as already mentioned, the generator is "milked" every 24 h to remove the ^{99m}Tc. Figure 2.3 also shows the corresponding dynamic change in Q_2 for a 7-day period.

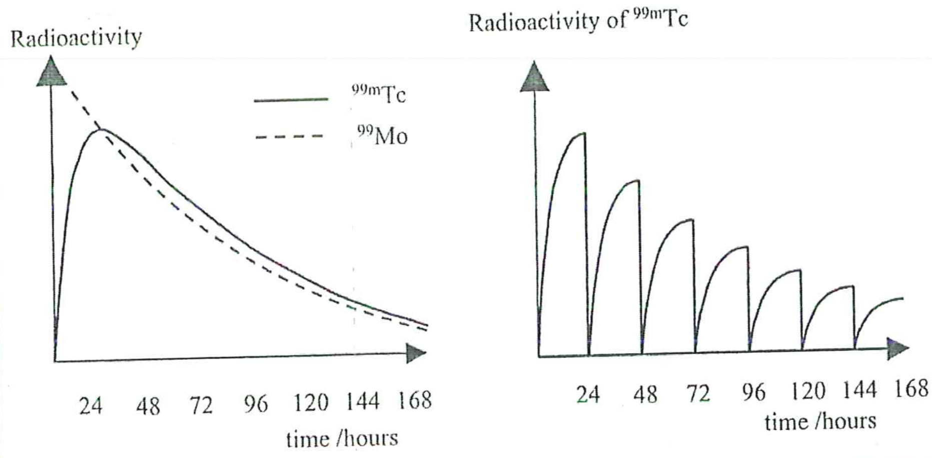


FIGURE 2.3. (Left) A plot of the radioactivity of ^{99}Mo and ^{99m}Tc as a function of time. (Right) A graph of the radioactivity of ^{99m}Tc in the technetium generator in the case where the ^{99m}Tc is removed every 24 h.

TABLE 3.1. Acoustic Properties of Biological Tissues

	Characteristic Acoustic Impedance $\times 10^5 (g\ cm^{-2}\ s^{-1})$	Speed of Sound $(m\ s^{-1})$
Air	0.0004	330
Blood	1.61	1550
Bone	7.8	3500
Fat	1.38	1450
Brain	1.58	1540
Muscle	1.7	1580
Vitreous humor (eye)	1.52	1520
Liver	1.65	1570
Kidney	1.62	1560

Sound wave reflection and transmission

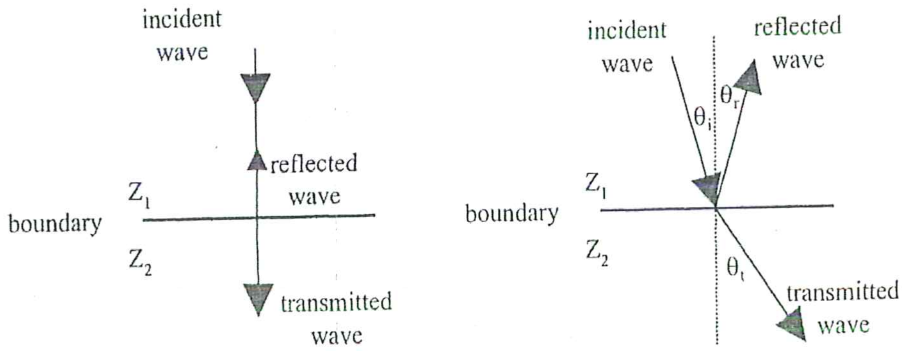


Fig 3.3. /Webb

In the case where the angle between the incident beam and boundary is not 90°, as shown on the right-hand side of Figure 3.3, the equations governing the angles of reflection and transmission are given by

$$\theta_i = \theta_r \tag{3.15}$$

$$\frac{\sin\theta_i}{\sin\theta_t} = \frac{c_1}{c_2} \quad \text{(eqv. to Snell's law in optics!)} \tag{3.16}$$

where c_1 and c_2 are the speed of sound in tissues 1 and 2, respectively. If the values of c_1 and c_2 are not equal, then the transmitted signal is refracted. This angular deviation from the original direction of propagation can cause misregistration artifacts in the image (Section 3.7). The pressure and intensity reflection and transmission coefficients are given by

$$R_p = \frac{p_r}{p_i} = \frac{Z_2\cos\theta_i - Z_1\cos\theta_t}{Z_2\cos\theta_i + Z_1\cos\theta_t} \tag{3.17}$$

$$T_p = \frac{p_t}{p_i} = \frac{2Z_2\cos\theta_i}{Z_2\cos\theta_i + Z_1\cos\theta_t} \tag{3.18}$$

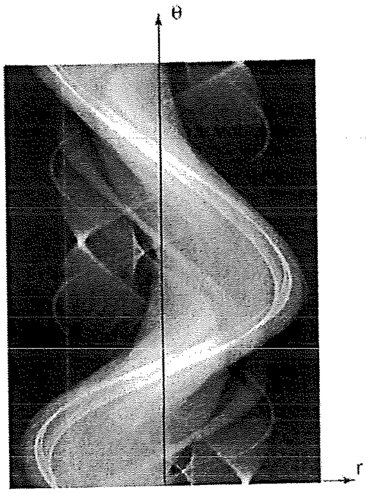
$$R_I = \frac{I_r}{I_i} = \frac{(Z_2\cos\theta_i - Z_1\cos\theta_t)^2}{(Z_2\cos\theta_i + Z_1\cos\theta_t)^2} \tag{3.19}$$

$$T_I = \frac{I_t}{I_i} = \frac{4Z_2Z_1\cos^2\theta_i}{(Z_2\cos\theta_i + Z_1\cos\theta_t)^2} \tag{3.20}$$

$$\frac{dM_x}{dt} = \gamma M_y\left(B_0 - \frac{\omega}{\gamma}\right) - \frac{M_x}{T_2}$$

$$\frac{dM_y}{dt} = \gamma M_z B_1 - \gamma M_x\left(B_0 - \frac{\omega}{\gamma}\right) - \frac{M_y}{T_2}$$

$$\frac{dM_z}{dt} = -\gamma M_y B_1 - \frac{M_z - M_0}{T_1}$$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