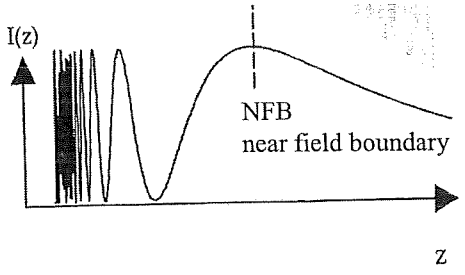
The Beam Geometry of a Single Transducer. The simplest transducer, termed a plane-piston, is one in which the piezoelectric crystal has a flat face. The properties of the transmitted ultrasound wave can be modeled by considering the transducer to be made up of a large number of point sources, each of which emits a spherical wave. The total pressure wave is a superposition of each of these individual components. If wave propagation is in the z direction, then the on-axis, or axial, intensity $I(z)$ of the wave is given by

$$I(z) \approx 2\rho c u_z^2 \sin^2\left[\frac{\pi}{2}\left(\frac{a^2/\lambda}{z}\right)\right] \tag{3.30}$$

where a is the radius of the crystal



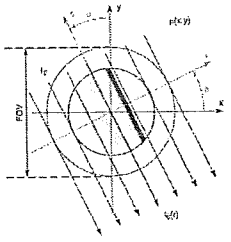
Construction of slice images from projections

One of the major developments in medical imaging over the past two decades has been the development of techniques for constructing images representing slices through three-dimensional objects. These techniques are called tomography (tomos = slice) and are based on the idea that an object may be constructed from *projections* of the object. That this was possible was first shown by Radon in 1917, but the method was not formally developed until the end of the 1960s.

5.3.1.1 Projection and Radon Transform

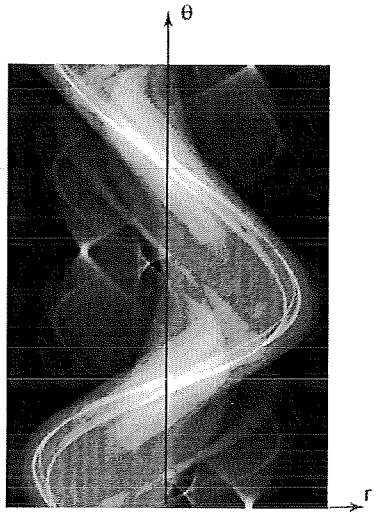
Consider the 2D parallel-beam geometry in Figure 5.5(a) in which $\mu(x, y)$ represents the distribution of the linear attenuation coefficient in the xy-plane. It is assumed that the patient lies along the z-axis and that $\mu(x, y)$ is zero outside a circular *field of view* with diameter FOV. The X-ray beams make an angle θ with the y-axis. The unattenuated intensity of the X-ray beams is I_0. A new coordinate system (r, s) is defined by rotating (x, y) over the angle θ. This gives the following transformation formulas:

$$\begin{bmatrix} r \\ s \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}, \quad \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} r \\ s \end{bmatrix}, \tag{5.2}$$



and the Jacobian is

$$J = \begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix} = 1. \tag{5.3}$$



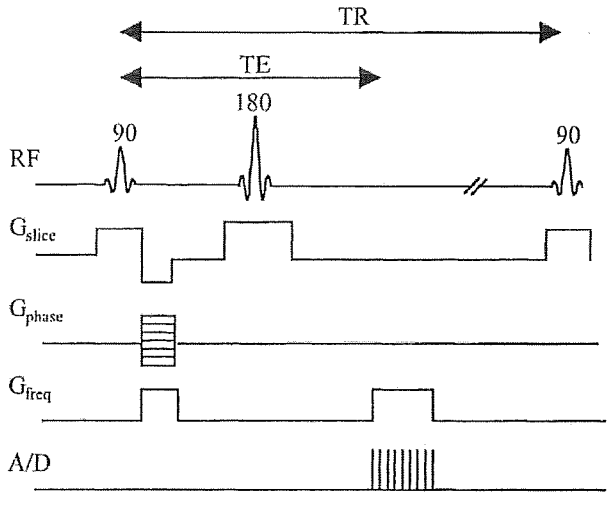


FIGURE 4.27. *The basic spin-echo imaging sequence. The values of the echo time (TE) and the repetition time (TR) between successive increments of the phase-encoding gradient can be adjusted to maximize tissue contrast.*

Instead of applying a negative dephasing gradient followed by a positive rephasing gradient, as for the gradient-echo sequence in Figure 4.20, the dephasing gradient in a spin-echo sequence is usually applied between the 90° and 180° pulses with a positive polarity. As in the spectroscopic sequence used to measure T_2, the 180° pulse reverses the phase accumulated by the protons from, in this case, the two positive-polarity gradients.

There are time periods in the imaging sequence when no gradients or pulses are applied. These delays are introduced to give certain values to the TR and TE in order to introduce corresponding T_1- and T_2-contrast weighting into the image, as discussed in the next section.

4.5.2. T_1- and T_2-Weighted Imaging Sequences

The intensity of an axial image acquired using a spin-echo sequence is given by

$$I(x, y) \propto \rho(x, y)\left(1 - e^{-\mathrm{TR}/T_1}\right) e^{-\mathrm{TE}/T_2} \tag{4.53}$$

where $I(x, y)$ is the pixel intensity at each point (x, y) and $\rho(x, y)$ is the "proton density," the number of protons at each point (x, y). The term $1 - \exp(-\mathrm{TR}/T_1)$ determines the "T_1-weighting" of the sequence, that is, the extent to which the image intensity is governed by the different T_1 values of the tissues. The value of TR can be set by the operator from the imaging console, and this value is chosen to give the best CNR between, for example, tumor and healthy tissue. If the value of TR is set to a value much greater than the T_1 of any of the tissues, then the image has no T_1-weighting because the term $1 - \exp(-\mathrm{TR}/T_1)$ is very close to unity for all tissues. If the value of TR is set closer to the tissue T_1 values, then the image becomes more T_1-weighted. The concept of T_1-weighting is shown in Figure 4.28, using values of T_1 from Table 4.2 for

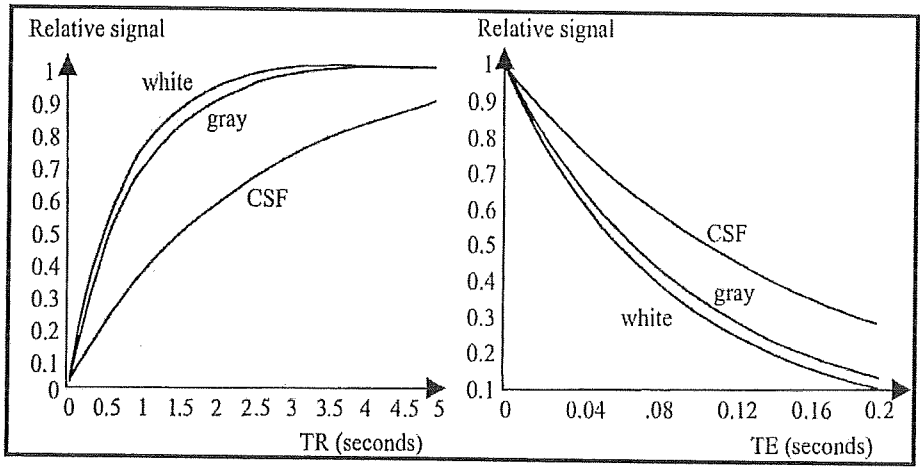


FIGURE 4.28. The effect of data acquisition parameters on the relative signal intensities from cerebrospinal fluid (CSF), white matter, and gray matter in the brain. (Left) The relationship between signal intensity and TR, showing increased T_1-weighting at shorter TR values. As TR becomes very long, the difference in the relative signals becomes smaller. (Right) A graph showing the corresponding relationship between signal intensity and TE. The highest T_2-weighting occurs at long values of TE, but the image intensities are low.

Tfy-99.4275 – Signal Processing in Biomedical Engineering

Exam 14.01.09 16:00-19:00

For each question a maximum of 6 points can be earned (thus: 5 * 6 = 30 points in total). Possible points from the home exercises will be added to these points.

You may answer the questions in English as well as in Finnish.

1.

Answer shortly to the following questions:

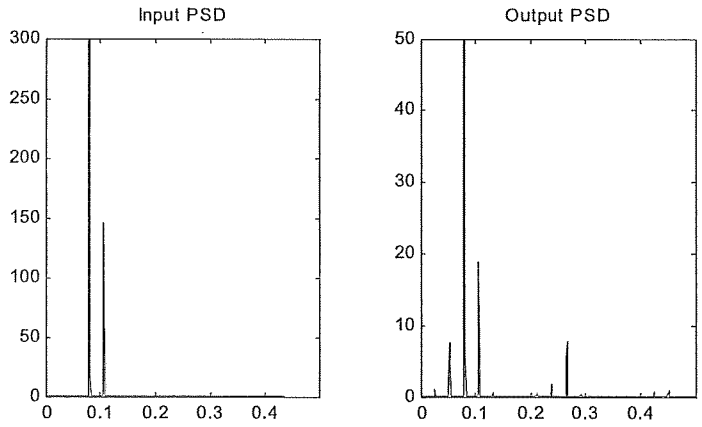
a) Stability and norms:

1 – Define the L_1, L_2 and L_∞-norms, and explain which features of the signal each of them emphasise, respectively. (2p)
2 – Define the term 'stability'. (1p)
3 – Suppose we have a linear system with impulse response $h(k)$, which is a rational function with zeros z_i and poles p_i. How do we know whether this linear system is stable or not? (1p)

b) Describe aliasing graphically in time- or in the frequency- domain. (2p)

2.

a) We have a system whose input PSD is provided on the left and the output PSD on the right. Is the system a linear time-invariant (LTI) system? Give arguments. (2p)



In autoregressive modelling a signal is represented as an output of a mathematical model.

b) Give a mathematical presentation of the autoregressive (AR) model. (2 p)
c) An AR model of a signal gives us the possibility to estimate the power spectral density (PSD) of the signal. However, to use that approach we need to check the validity of some assumptions; give two of them. (2 p)
