

Final Examination. | 8.1.2007

Please write on sheet on separate lines:

- 1) course title, date
- 2) student number + letter, IN BLOCK LETTERS surname underlined, all given names
- 3) signature

1. Evaluate each of the following limit, if it exists

(a) $\lim_{(x,y) \rightarrow (0,0)} x \ln(|x| + |y|)$

2. (b) $\lim_{(x,y,z) \rightarrow (0,0,0)} \frac{\sin \sqrt{x^2+y^2+z^2}}{x^2+y^2+z^2}$

Evaluate the double integral $I_2 = \iint_G \frac{\ln(x^2+y^2)}{x^2+y^2} dA$, where $G = \{1 \leq x^2 + y^2 \leq a^2, y \geq 0\}$

3. Find the maximum and minimum values of $f(x, y, z) = xy + z^2$ on the ball $x^2 + y^2 + z^2 \leq 1$. Use Lagrange multipliers to treat the boundary case.

4. (a) Solve the initial value problem

$$2y'' + 5y' - 3y = 0, \quad y(0) = 1, \quad y'(0) = 0$$

(b) Determine whether the following series converge absolutely, converge conditionally, or diverge

i. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2 + \ln n}$

ii. $\sum_{n=1}^{\infty} \frac{(-2)^n}{n!}$

iii. $\sum_{n=1}^{\infty} \frac{100 \cos(n\pi)}{2n+3}$

5. Find the flux of the vector field $\vec{F} = e^{-y}\vec{i} - y\vec{j} + x \sin z\vec{k}$, across σ in the direction of the positive orientation, where σ is the portion of the elliptic cylinder $\vec{r}(u, v) = 2 \cos v\vec{i} + \sin v\vec{j} + u\vec{k}$ with $0 \leq u \leq 5$ and $0 \leq v \leq 2\pi$.