# T-61.3040 Statistical Modeling of Signals

Final Exam 15.12.2008

In the exam you are allowed to have a calculator (non-programmable or memory emptied) and basic mathematical tables (no tables containing material directly associated with the course). The results of the exam will be announced eventually through the Noppa system, and not anymore through the newsgroup opinnot.tik.informaatiotekniikka.

**1. (max 6p)**

Explain *briefly* the following topics without unnecessary detail:

i) Ergodicity (2p)

ii) Pisarenko's method (2p)

iii) Wold decomposition (2p)

**2. (max 6p)**

You have observed the real-valued zero-mean WSS process $x(n)$ and you know the following values of the autocorrelation: $r_x(0) = 1$, $r_x(1) = 2/3$, $r_x(2) = 0$, $r_x(3) = 1/3$, $r_x(4) = 0$.

i) Form an optimal linear predictor having two weights, where you predict the value $x(n)$ using the observations $x(n-1)$ and $x(n-2)$. (2p)

ii) What is the mean-square prediction error of your predictor? How much smaller is the prediction error compared to the variance $\mathrm{Var}(x(n))$ of the process? (2p)

iii) Can you improve the prediction accuracy using a linear predictor with two weights, where you predict the value $x(n)$ using observations $x(n-2)$,$x(n-3)$ or observations $x(n-3)$,$x(n-4)$? (2p)

**3. (max 6p)**

Answer the following propositions either "true" or "false": you may also leave any of them unanswered. A correct answer gives 1 point, a wrong answer $-1$ points, and a missing answer zero points. However, the total number of points you receive from this problem cannot become negative; the total number of points is at least zero. No need to justify your answers.

a) If $x(n) = d(n) + v(n)$, $x(n)$ and the desired signal $d(n)$ are jointly WSS, and $d(n)$ and the white noise $v(n)$ are uncorrelated, then based on these facts it is possible to give an upper limit to the frequency response of an IIR Wiener filter.

b) An MA(q) process has infinitely many nonzero autocorrelations $r_x(k)$.

c) The autocorrelation $r_y(n)$ of the output is the convolution of the autocorrelation $r_x(n)$ of the input and the unit sample response $h(n)$.

d) If the process $x(n)$ and the desired process $d(n)$ are jointly WSS and the step size of the LMS algorithm satisfies $0 < \mu < 2/\lambda_{max}$ (where $\lambda_{max}$ is the maximum eigenvalue of the matrix $\mathbf{R}_x$), then the mean square error of the LMS algorithm converges to the mean square error of the corresponding Wiener filter.

e) If we estimate a $M \times M$ autocorrelation matrix from $N$ observations, the covariance method computes each of its estimates from at least equally many observations as the autocorrelation method.

f) The ARMA(2,1) process $x(n) = x(n-1) - (1/4)x(n-2) + v(n) + (1/2)v(n-1)$, where $v(n)$ is normally distributed white noise with variance 1, is wide sense stationary.

**4. (max 6p)**

You have measured the following three observations from the real-valued zero-mean WSS process $x(n)$: $x(0) = 2$, $x(1) = 1$, $x(2) = 1$. You know that the process should have a lot of power in one frequency, which is either $\pi/3$ or $\pi/2$.

i) Estimate a $3 \times 3$ autocorrelation matrix for the process so that the result is positive semidefinite. (1.5p)

ii) Estimate the value of the power spectrum for the two above-mentioned frequencies using the periodogram. Based on the estimates, which frequency is more likely to be correct? (1.5p)

iii) Estimate the value of the power spectrum for the two above-mentioned frequencies or compute the value of the pseudospectrum for the frequencies, using a parametric method of your choice. Based on the results, which frequency is more likely to be correct? You can use a smaller $2 \times 2$ autocorrelation matrix if the computation in your chosen method is otherwise too difficult. (3p)

Possibly useful information:

$\sin(\frac{\pi}{3}) = \frac{\sqrt{3}}{2}$, $\cos(\frac{\pi}{3}) = \frac{1}{2}$, $\sin(\frac{2\pi}{3}) = \frac{\sqrt{3}}{2}$, $\cos(\frac{2\pi}{3}) = -\frac{1}{2}$.

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