Tfy-0.3223 Statistical Physics Intermediate exam / välikoe Wed March 11, 2009

Answer to four (4) questions of your choice. / Vastaa valitsemaasi neljään kysymykseen.

1. *Classical harmonic oscillators*: consider N harmonic oscillators with coordinates and momenta $\{q_i, p_i\}$, and the Hamiltonian

$$H(\{q_i, p_i\}) = \sum_{i=1}^{N} \left[\frac{p_i^2}{2m} + \frac{m\omega^2 q_i^2}{2} \right]. \quad (1)$$

(a) Calculate the entropy S as a function of the total energy E. (Hint: transform the surface of equal energy into a d-dimensional sphere by a change of variables.) You can use the expression $V_d = \frac{2\pi^{d/2}R^d}{(d/2-1)!d}$ for the volume of a d-dimensional ball with radius R.

(b) Calculate the energy E and heat capacity C as functions of temperature T and N.

(c) Find the joint probability density $P(p, q)$ for a single oscillator. Use it to calculate the mean kinetic energy and mean potential energy for each oscillator.

2. *Quantum rotor:* consider a rotor in two dimensions with the Hamiltonian

$$H = -\frac{\hbar^2}{2I}\frac{d^2}{d\theta^2}, 0 \leq \theta < 2\pi. \quad (2)$$

(a) Find the energy levels and eigenstates.

(b) Write the expression for the density matrix $\langle\theta'|\rho|\theta\rangle$ in a canonical ensemble at temperature T, evaluate both the low- and high-temperature limits.

3. *Random variables:* start with a random variable with an exponential probability distribution function $p(x) = a\exp(-ax)$.

(a) Add two variables (independent) together. Compute the resulting distribution.

(b) Repeat the above, but for N variables.

(c) Give a reasonable approximation for the previous result for N variables for N large.

4. *Kinetic theory:*

(a) What is the H-theorem?

(b) What does it tell about approaching the equilibrium?

(c) What kind of equilibrium distribution is indicated (for an ideal gas)?

5. *Quantum statistics and gases:*

(a) What does the classical heat capacity of a polyatomic gas depend on, and how does the quantization of vibrations influence the result and when?

(b) The flux of emitted radiation from a black body is given by $I(k,T)dk$ where $I = (\hbar c^2/4\pi^2)k^3/(e^{(\beta\hbar ck)} - 1)$. Explain the consequences.

(c) Explain the heat capacity of a Fermi gas at low temperatures. Draw it roughly vs. temperature.