

# T-61.5130 Machine Learning and Neural Networks

## Examination 18th December 2008/Karhunen

You are allowed to have in the examination a collection of mathematical formulas, but not any of the teaching material. (Voit vastata tenttiin myös suomeksi.)

1. Answer briefly (using a few lines) to the following questions or items:

- (a) For what purpose and where is moment term used?
- (b) What is cross-validation?
- (c) Which are the two main criteria for measuring non-Gaussianity?
- (d) Explain briefly  $\epsilon$ -insensitive cost function.
- (e) Which neural network method is based on competitive learning?
- (f) Explain briefly what is NARX model.

2. Let the error function be

$$\mathcal{E}(\mathbf{w}) = w_1^2 + 10w_2^2,$$

where  $w_1$  and  $w_2$  are the components of the two-dimensional parameter vector  $\mathbf{w}$ . Find the minimum value of  $\mathcal{E}(\mathbf{w})$  by applying the steepest descent method. Use  $\mathbf{w}(0) = [1, 1]^T$  as an initial value for the parameter vector and the following constant values for the learning rate:

- (a)  $\alpha = 0.04$
  - (b)  $\alpha = 0.1$
  - (c)  $\alpha = 0.2$
  - (d) What is the condition for the convergence of this method?
3. Consider solving the XOR problem using an RBF network. In the XOR problem the desired output is 0 for the vectors  $(1, 1)^T$  and  $(0, 0)^T$  belonging to the first class. For the vectors  $(1, 0)^T$  and  $(0, 1)^T$  belonging to the second class the desired output is 1. Let us construct a classifier using the basic RBF network, where the radial basis functions are chosen to be multiquadratic type functions

$$\varphi(\|\mathbf{x} - \mathbf{x}_i\|) = [\|\mathbf{x} - \mathbf{x}_i\|^2 + 3]^{1/2}$$

where  $\mathbf{x}_i$  is the  $i$ :th training vector. Present how the solution is calculated, and form the equations needed for the solution. Note: you do not need to solve these equations numerically.

4. Consider  $n$  zero mean source signals  $s_i$ , which have been collected to the vector  $\mathbf{s} = (s_1, s_2, \dots, s_n)^T$ . Assume that the joint distribution of the sources is  $n$  variables multivariate Gaussian distribution

$$p(\mathbf{s}) = (2\pi)^{-n/2} (\det \mathbf{C})^{-1/2} \exp \left\{ -\frac{1}{2} \mathbf{s}^T \mathbf{C}^{-1} \mathbf{s} \right\}$$

CONTINUES ON THE REVERSE SIDE →

Here  $\mathbf{C} = E\{\mathbf{s}\mathbf{s}^T\}$  is the covariance matrix of the vector  $\mathbf{s}$  and  $\det$  means determinant. Assume that the source signals  $s_1, s_2, \dots, s_n$  are mutually uncorrelated. Show that they are mutually statistically independent.

Does this property hold for other than Gaussian distributed source signals? How about the reverse property: are statistically independent source signals source signals mutually uncorrelated?