## Mat-2.103 Design of experiments

Heliövaara
Final exam 24.4. 2006

> Write on each paper:
> - Mat-2.103 Final exam 24.4. 2006
> - student number and the letter
> - IN A CLEAR WAY surname and all forenames
> - study program and year
> - possible previous names and study programs
> - signature

1. Explain the following terms shortly
a) Orthogonal contrasts.
b) Significance level.
c) Interaction of two factors.
d) Bartlett's test.
e) Least-squares method in parameter estimation.
f) Nuisance variable (Extraneous variable).
2. a) The model for one-factor analysis of variance (one-way classification fixed-effects model) can be expressed in three ways. Present two of the models and the null hypothesis relating to them.
b) In the figures below are presented three graphs of means for the four combinations of levels of factors $A$ and $B$.

a)

b)

c)

Does there appear $A$-effect, $B$-effect, interaction effect $A B$ ? Explain your answers shortly for each graph.
3. The influence of factors $A$ and $B$ to response variable $Y$ are examined by conducting a $2^{2}$-factorial experiment with two replications for each combination. The results:

| $A$ | $B$ | $Y$ |  |
| :---: | :---: | :---: | :---: |
| - | - | 4.0 | 3.5 |
| + | - | 7.5 | 9.0 |
| - | + | 5.5 | 7.0 |
| + | + | 13.0 | 16.0 |

Is there interaction between factors $A$ and $B$ ? Do the test using significance level 0.05.
4. Three detergents were compared by washing three containers each once with each detergent. After the wash the number of bacteria in the container was measured. The washing was performed by three persons. The results:

| detergent | container | washer | result |
| :---: | :---: | :---: | :---: |
| $C$ | 1 | 1 | 5.8 |
| $A$ | 2 | 1 | 6.4 |
| $B$ | 3 | 1 | 5.6 |
| $B$ | 1 | 2 | 5.5 |
| $C$ | 2 | 2 | 5.5 |
| $A$ | 3 | 2 | 5.7 |
| $A$ | 1 | 3 | 6.2 |
| $B$ | 2 | 3 | 5.7 |
| $C$ | 3 | 3 | 5.2 |

Is there difference in the effectiveness of the detergents. Do the test with significance level 0.05 . The variations between the containers and the washers have to be eliminated. What is the name of the design used?

The sum of squares of the observed values =

$$
\sum_{i=1}^{3} \sum_{j=1}^{3} \sum_{k=1}^{3} y_{k i j}^{2}=296.92
$$

5. A null hypothesis $H_{0}: \mu=4.5$ is tested with an alternative hypothesis $H_{1}: \mu \neq 4.5$, where $\mu$ is the mean of a normally distributed random variable $X$. It is known that the standard deviation of $X$ is $\sigma=0.26$. The significance level of the test is $\alpha=0.05$ and the null hypothesis is tested by determining the arithmetic average $\bar{x}$ of 12 observations.
a) Which values of $\bar{x}$ result to rejecting $H_{0}$ ?
b) What is the power of the test when $\mu=4.6$ ? (i.e. what is the probability $H_{0}$ is rejected when $\mu=4.6$ ?)
