Helsinki University of Technology
Laboratory for Theoretical Computer Science
Harri Haanpää (tel. 5243)

T-79.1002 Introduction to Theoretical Computer Science Y (2 ECTS)
Exam Thu 20 December 2007, 1–4 p.m.

Write down on each answer sheet:
- Your name, department, and student id
- The text: "T-79.1002 Introduction to Theoretical Computer Science Y 20.12.2007 "
- The total number of answer sheets you are submitting for grading

1. Describe each of the following languages as a finite automaton.

   (a) $\{w \in \{0,1\}^* \mid |w| \geq 2, |w| \text{ is even}\}$, *3p.*

   (b) $\{w \in \{0,1\}^* \mid |w| \geq 3, w \text{ starts with } 110 \text{ or ends with } 010\}$, *3p.*

   (c) $\{w \in \{a,b,c\}^* \mid w \text{ contains neither } aa \text{ nor } bc \text{ as a substring}\}$. *4p.*

2. Describe each of the following languages as a regular expression.

   (a) $\{w \in \{0,1\}^* \mid |w| \geq 2, |w| \text{ is even}\}$, *3p.*

   (b) $\{w \in \{0,1\}^* \mid |w| \geq 3, w \text{ starts with } 110 \text{ or ends with } 010\}$, *3p.*

   (c) $\{w \in \{a,b,c\}^* \mid w \text{ contains } aa \text{ or } bc \text{ as a substring}\}$. *4p.*

3. Consider strings over the alphabet $\{0,1\}$. Let $n_0(w)$ denote the number of 0s in the string w. Let

   $$L_1 = \{0^i 1^j \mid i > j \geq 0\}$$

   and

   $$L_2 = \{w \mid n_0(w) \leq 3\}.$$

   Give context-free grammars that produce L_1 and $L_1 \cup L_2$. *10p.*

4. Justify the claim: if language $L \subseteq \{a,b,c\}^*$ can be recognized by a finite automaton, then so can language $L' \subseteq \{a,b\}^*$, that is obtained by replacing each c in the strings of L with the string ab. *10p.*

Total 40p.