

Mat-1.1610 Mathematics 1

1st mid-term exam 13.10.2009, 16–19.

Calculators without symbolic simplification properties are allowed.

1. Find the polar representations (modulus and argument) of the complex numbers

$$\frac{3-3i}{1+i} \quad \text{and} \quad \frac{1}{\cos x + i \sin x}.$$

Here x is a real number.

2. Let $u = (1, 1, 1) = \mathbf{i} + \mathbf{j} + \mathbf{k}$. Find a representation $u = v + w$, where v is parallel to $(1, -2, 2) = \mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$ and w is orthogonal to v , $w \perp v$.
3. A plane passes through the points $(1, -1, 1)$, $(3, 1, 2)$ and $(-1, 2, 5)$. Find the equation of the plane in the form $ax + by + cz = d$.
4. a) For which values of the parameter $t \in \mathbf{R}$ does the matrix

$$[A] = \begin{bmatrix} 1 & 1 & t \\ 1 & 2 & 1 \\ 1 & 3 & 1 \end{bmatrix}$$

have an inverse $[A]^{-1}$?

b) Show that the inverse exists for $t = 2$ and calculate $[A]^{-1}$ in this case.

c) Solve the system $[A]^2 \mathbf{x} = [0, 0, 1]^T$ by using the inverse of part b. Here $[A]^2 = [A][A]$.