

Teknillinen korkeakoulu

Mat-1.1110 Matematiikan peruskurssi C1

Hakula/Byckling

1. välikoe, syksy 2009, 13.10.2009

Calculators are permitted. Every problem is worth six points. In multi-part problems every part carries an equal weight, unless otherwise indicated.

Problems

1. Find the inverse(s) of a (if possible) in the given sets

a) $a = 13 + 10i; \mathbb{C}$ b) $a = 3; \mathbb{Z}_7$ c) $a = 2009; \mathbb{Z}_8$.

2. Let

$$A = \{(a, b) \mid a, b \in \mathbb{Z}, b \neq 0\} \subset \mathbb{Z} \times \mathbb{Z}$$

and relation \sim in A such that $(a, b) \sim (c, d)$, if and only if $ad = bc$. Show that \sim is an equivalence relation.

3. Solve the congruence equation

$$1310x \equiv 2009 \pmod{2011}.$$

4. Let $G = (V, E)$ be a graph with $2k$ nodes ($k \in \mathbb{N}$) and without a complete graph K_3 as subgraph. Show using induction that the number of edges of G is at most k^2 . Give an example of a graph where the upper bound is realised.