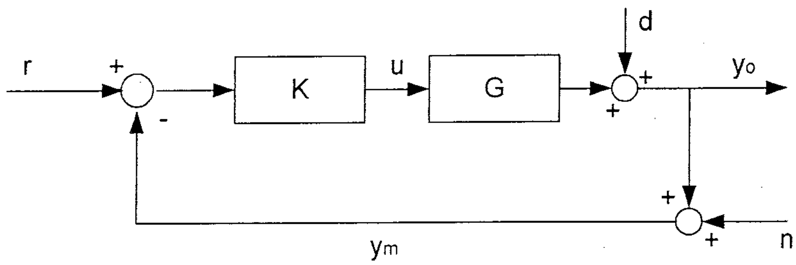
AS-74. 3123 Model-Based Control Systems
Exam 17. 12. 2009

The questions are available only in English. You can answer in Finnish, Swedish or English. The final grade is given when both the examination and the homework problem have been accepted.

5 problems.

1. Consider the multivariable closed-loop control configuration shown in the figure. Let the dimension of matrix G be p x k.



Identify

a. the dimensions of matrix K and all signals shown in the figure.
b. closed-loop transfer function
c. sensitivity function
d. complementary sensitivity function

Write expressions for the output variable y_0 , control variable u and error variable $e = r - y_0$, as functions of the input variables r, d and n.

2. Explain briefly the following concepts

a. singular values
b. H_∞-norm
c. internal stability $w \to u$, $w \to y$ in $w_0 \to u$ $w_0 \to y$ for stable
d. Smith predictor
e. Small gain theorem
f. "Internal model control" (IMC)

3. a. Let A and B be such matrices that the multiplications AB and BA are both properly defined. Prove that it holds

$$A(I+BA)^{-1}=(I+AB)^{-1}A$$

This expression has a particular name. What is it?

b. Explain shortly what is meant by *dynamic programming* and the *principle of optimality*?

c. Let S denote the sensitivity function of a controlled system and

$$M_S=\max_{\omega}|S(j\omega)|$$

Draw a typical Nyquist diagram of the loop transfer function L and show geometrically the meaning of M_S.

4. Consider the system

$$\begin{cases}\dot{x}_1=-x_1+u, & x_1(0)=x_{10}\\ \dot{x}_2=x_1, & x_2(0)=x_{20}\end{cases}$$

and the criterion to be minimized

$$J=\int_0^{\infty}\left(x_2^2+au^2\right)dt \qquad (a>0,\text{ constant})$$

Determine the optimal control law. What is the minimum value of J then?

Note: You do not have to solve the Riccati equation (although it could be done analytically). It is enough that you write each "component equation" separately, and name the solutions s_{11}, s_{12} etc, to be used later in the problem.

5. Consider a multivariable system with the transfer function matrix

$$G(s)=\begin{bmatrix}\dfrac{2}{s+1} & \dfrac{3}{s+2}\\ \dfrac{1}{s+1} & \dfrac{1}{s+1}\end{bmatrix}$$

a. Draw a "Simulink like" diagram of the closed loop, when the system is controlled by two separate PI controllers. Explain in this context what "pairing problem" in multivariable control means. (In the diagram you can plot any transfer function in one block. Do not divide transfer functions into smaller pieces.)

~~4~~6. Explain how you would use RGA analysis, SVD and decoupling to control the above system.

Some formulas, which might be useful:

$$\dot{x} = Ax + Bu, \quad t \geq t_0$$

$$J(t_0) = \frac{1}{2}x^T(t_f)S(t_f)x(t_f) + \frac{1}{2}\int_{t_0}^{t_f}\left(x^T Qx + u^T Ru\right)\ dt$$

$$S(t_f) \geq 0, \quad Q \geq 0, \quad R > 0$$

$$-\dot{S}(t) = A^T S + SA - SBR^{-1}B^T S + Q,\ t \leq t_f, \text{ boundary condition } S(t_f)$$

$$K = R^{-1}B^T S$$

$$u = -Kx$$

$$J^*(t_0) = \frac{1}{2}x^T(t_0)S(t_0)x(t_0)$$

$A .\!* (A^{+})^{T}$

$A = U\Sigma V^*$

$V \hat{=} A^*A$

$U \hat{=} AA^*$

$\int_0^\infty \log|S| = \sum_i \mathrm{Re}\, p_i$

i counted with RHP(G)

$p_i \hat{=} L = GF_1$

$L = (1 + GF_1)^{-1}$