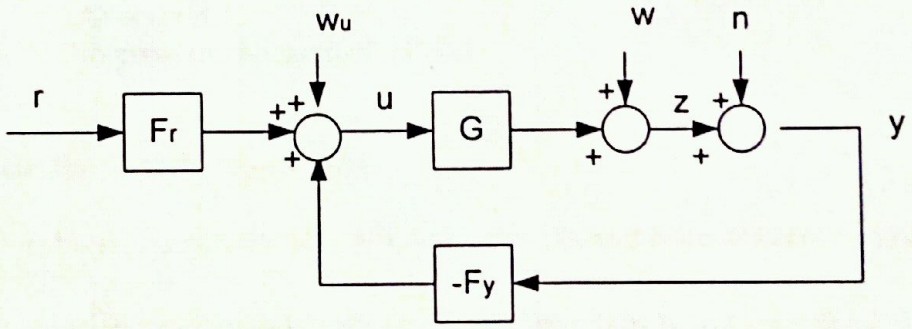
AS-74. 3123 Model-Based Control Systems
Exam 15. 12. 2011

The questions are available only in English. You can answer in Finnish, Swedish or English. The final grade is given when both the examination and the homework problem have been evaluated and accepted.

5 problems.

1. Consider a linear multivariable (MIMO) system in the 2 DOF configuration shown in the figure. Assume that $w_u = 0$.



a. Explain the terms *one degree-of-freedom control configuration*, *two degrees-of-freedom control configuration* and *generalized control configuration*? Illustrate with figures.

b. Derive the following formulas and name the symbols used. (In the derivations remember that this is a MIMO system).

$$z = G_c r + Sw - Tn$$
$$e = (I - G_c)r - Sw + Tn$$

c. Prove that for all frequencies it holds

$$S + T = I$$
$$(I + GF_y)^{-1} GF_y = I - (I + GF_y)^{-1}$$
$$T = (I + (GF_y)^{-1})^{-1}$$

2 a. Explain shortly what is meant by *dynamic programming* and the *principle of optimality*?

2b. Consider a SISO-system in one degree-of-freedom configuration. The ralationship between the real and nominal system is

$$G_0(s) = G(s)(1 + \Delta_G(s))$$

where the variables represent standard symbols used in the course.

Derive an expression stating that the system is robustly stable. Explain verbally what robust stability means.

3. Explain briefly the following concepts

- **a.** singular values
- **b.** H_∞-norm
- **c.** internal stability
- **d.** "Relative gain array" (RGA)
- **e.** LQ-control
- **f.** "Internal model control" (IMC)

4. Consider the discrete-time system

$$x(k+1) = ax(k) + bu(k),\ x(0) = x_0 \qquad (a \text{ and } b \text{ are nonzero constants})$$

By using dynamic programming calculate the controls $u(k)$, $k = 0,1$, which minimize the criterion

$$J = \sum_{i=0}^{1}\left[x(i)^2 + u(i)^2\right]$$

Consider the cases where the final state is i. *free* and ii. *fixed*, $x(2) = 0$. Calculate the optimal costs also.

5. Consider the following formulas

$$\int_0^\infty \log\left|S(i\omega)\right| d\omega = \pi \sum_{i=1}^{M} \mathrm{Re}\,(p_i)$$

$$|W_T(p_1)| \le 1 \quad \Rightarrow \quad \omega_0 \ge \frac{p_1}{1 - 1/T_0}$$

$$|W_S(z)| \le 1 \quad \Rightarrow \quad \omega_0 \le (1 - 1/S_0)\,z$$

where the symbols are standard symbols used in the course. Explain shortly (without any derivations) the meaning of the formulas from the viewpoint of control design.