HELSINKI UNIVERSITY OF TECHNOLOGY **Exam 15 January 2010**

Signal Processing Laboratory Stefan Werner

S-88.2111 Signal Processing in Telecommunications I (3 cp)

Write in each answer paper your name, department, student number, the course name and the date. Number each paper you submit and denote the total no. of pages. The exam paper is only available in English. 5 problems, 30 points total. The BETA mathematical tables *can be utilized (you can borrow a copy from the exam supervisor if needed). Also a* basic calculator *can be used (no memory, no graphics).*
The homework bonus will be valid for possible future exams too.

1. Define and describe *briefly* (2..3 lines of text) the following concepts (1p each):
   a) Viterbi equalizer
   b) ZF equalizer
   c) MSEG algorithm
   d) Roll-off factor
   e) Nyquist criterion
   f) Channel capacity

2. (6p) Echo cancellers are used in many wireline transmission systems.

   (3p) Explain the motivation and the basic idea of the echo canceller with block diagrams etc.

   (3p) Show how an adaptive filter can be used to implement the echo canceller filter. Use block diagrams, introduce the necessary notation and explain how the adaptive echo canceller operates.

3. DFE (7p total! Extra bonus point possible!)

   Consider a linear discrete-time channel model $C(z) = 1/(1+kz^{-1})$ where k is constant. We want to design a decision-feedback equalizer. For the design we assume a linearized model, i.e., all the decisions are correct. For simplicity, let us assume that the transmit filter response is included in the channel model (i.e., $H_T(z) = 1$).

   a) (2p) Derive the condition for optimal zero-forcing (ZF) linear equalizer for general z-transfer functions ($H_T(z)$, $H_R(z)$, $C(z)$, $F_R(z)$) using the linearized model. Draw the block diagrams of both the implementation structure and the linearized model.

   b) (2p) Design an optimal zero-forcing (ZF) linear equalizer $H_R(z)$, assuming $F_R(z) = 0$ and using the given channel $C(z) = 1/(1+kz^{-1})$.

   c) (2p) Design an optimal zero-forcing decision-feedback (DFE) equalizer $F_R(z)$ assuming that $H_R(z) = 1$ and using the given channel $C(z) = 1/(1+kz^{-1})$.

(Please turn for more questions)

4. (6p) Matched filters.

Consider a discrete-time receive filter $h_R(k)$ and its frequency response $H_R(e^{j\omega k})$. Assume a simple discrete-time transmit filter:

$$h_T(k) = 3\delta(k) - 2\delta(k+1) + \delta(k-2) \tag{1}$$

a) (3p) Find the matched-filter receive filter $h_R(k)$ and draw the impulse responses $h_T(k)$ and $h_R(k)$, both the ideal *noncausal* and *causal* versions.

b) (3p) Determine the pulse waveform $g(k)$ at the output of the receive filter either via convolution:

$$g(k) = h_R(k) * h_T(k) = \sum_{l=-\infty}^{\infty} h_R(l) h_T(k-l) \tag{2}$$

or in the frequency domain if you prefer. Plot $g(k)$.

5. Channel capacity (8 p)

In wireline communications, the channel does usually not vary with time but it is frequency dependent. Let us assume that the double-sided channel bandwidth is 2W (baseband channel) and the response is a piecewise constant with frequency but depends on the wireline length L in kilometers, as defined by Eq. (3). The channel noise is AWGN with the double-sided power spectrum $S_n(f) = P_n/(2W)$. $W = 3.8$ kHz and $SNR = P_x/P_n$ is 35 dB.

$$C(f) = \begin{cases} 1, & 0 < |f| < W/2 \\ 1/\sqrt{L}, & W/2 < |f| < 3W/4 \\ 1/2\sqrt{L}, & 3W/4 < |f| < W \end{cases} \tag{3}$$

a) (4p) Solve for the optimum transmit power spectrum $S_x(f)$ that maximizes the channel capacity when the total transmit power P_x is limited. Draw $S_x(f)$.

b) (4p) Determine the general formula (as a function of L) for this channel's capacity in a simple and compact form. Calculate the numerical values when the channel length is 100m and 200m.

Hints: The optimal power spectrum is obtained with the water-pouring theorem as

$$S_{x,opt}(f) = K - S_n(f)/|C(f)|^2 \tag{4}$$

where the Lagrange multiplier L is determined so that the total transmit power is limited to a constant value. The optimal capacity is then obtained by (double-sided) integration:

$$C = \frac{1}{2}\int_{-\infty}^{\infty} \log_2\left(1 + \frac{S_{x,opt}(f)|C(f)|^2}{S_n(f)}\right) df \tag{5}$$

(Please turn for more questions)