HELSINKI UNIVERSITY OF TECHNOLOGY
Department of Communications and Networking
S-38.3143 Queueing Theory

Examination
6.3.2008
Virtamo

Mark clearly (A/B) whetеher you would like to have the course registered A) with the new code 38.3143 (5 ETCS pt) or B) old code 38.143 (3 cr).

1. Let X be an exponential random variable. Without any computations, tell which one of the following is correct. Explain your answer.

   a) $\mathrm{E}[X^2|X>1] = \mathrm{E}[(X+1)^2]$

   b) $\mathrm{E}[X^2|X>1] = \mathrm{E}[X^2]+1$

   c) $\mathrm{E}[X^2|X>1] = (1+\mathrm{E}[X])^2$

2. There is a sliding door at the entrance to a supermarket. When a customer arrives in front of a closed door it takes S seconds before the door opens. During that time a queue builds up. When the door opens all the customers in the queue enter simultaneously. A timer closing the door is reset every time a customer goes in, and the door is closed first if during T seconds no one goes in. S and T are given constants. The interarrival times between the customers are assumed to obey the Exp(λ) distribution.

   a) How many customers on the average go through the door from the time it opens to the time it closes?

   b) What is the probability that an arriving customer has to wait in the queue?

3. a) Explain what system the Erlang formula relates to and under which assumptions it is valid.

   b) Derive the Erlang formula.

   c) How can the expression in the Erlang formula (Erlang's function) be computed recursively? Compute E(4,4).

4. Carloads of customers arrive at a single-server station according to a Poisson process with rate 4 per hour. The service time of each customer is exponentially distributed with mean 3 min. If each carload contains either 1, 2, or 3 customers with respective probabilities $\frac{1}{4}$, $\frac{1}{2}$, $\frac{1}{4}$, compute the average customer waiting time in the queue. Hint: The waiting time of the first customer of each group can be obtained from an appropriate $M/G/1$ queue. Consider separately the "internal" waiting time in the group.

5. A symmetric closed queuing network consists of 3 nodes (queues) with each of them having the service rate $\mu = 1\mathrm{s}^{-1}$. A customer departing a queue enters one of the other queues with equal probabilities. There are $K = 10$ customers in the network. Find the average sojourn time $\overline{T}$ of a customer in a node and the rate of the customer flow λ through each queue.