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EXAM
11.3.2009

S-72.3410 Coding Methods

1. (a) (3p.) A fountain code is used to transmit four packets (two bits each) of data, B_1, B_2, B_3, and B_4. Assume that the transmitted packets are B_1+B_2, B_2+B_3, B_3+B_4, B_1+B_4, $B_1+B_3+B_4$, and that the receiver gets the packets 00, 01, 01, 00, and 11 (in the same order). Determine the original packets B_i, $1 \leq i \leq 4$.

   (b) (3p.) Describe how the idea of network coding can be used to improve the rate when transmitting information between two ground stations via a satellite.

2. For the rate-1/2 convolutional encoder with $\mathrm{G}(D) = \begin{bmatrix} 1+D^2+D^3 & 1+D+D^3 \end{bmatrix}$

   (a) (1p.) Draw a hardware realization of the encoder.

   (b) (2p.) Draw the state diagram. Label the branches of the state diagram with input/output values.

   (c) (1p.) What is the constraint length of this code?

   (d) (2p.) What is d_{free} in this case? Justify your answer.

3. (a) (3p.) The Hamming bound can be written in the form

$$M \leq \frac{q^n}{V_q(n,t)}.$$

   What do M and $V_q(n,t)$ stand for here? Give an expression for $V_q(n,t)$ when $q=2$. Give a geometric interpretation for the Hamming bound.

   (b) (2p.) A block code is said to be *perfect* if it satisfies the Hamming bound with equality. Show that the binary Hamming codes are perfect.

   (c) (1p.) The Reed-Solomon codes are maximum distance separable (MDS). What does that mean?

4. Consider a binary, narrow-sense, 3-error-correcting BCH code of length 15.

   (a) (3p.) Find a generator polynomial for this code. You may find the following information helpful. If α is a primitive element in GF(16), here are the minimal polynomials of some powers of α with respect to GF(2):

α^1	x^4+x+1
α^3	$x^4+x^3+x^2+x+1$
α^5	x^2+x+1
α^7	x^4+x^3+1

   (b) (1p.) Determine the rate of the code.

   (c) (2p.) Construct a generator matrix and a parity-check matrix for this code.