(2)

Mat-2.114 Investment Science **Mild**

Examination 5.1.2006

Please write the following information on each answer sheet:

- Mat-2.114 Investment Science
- student registration number, surname and all first names
- study programme and year of studies
- signature

1. Please briefly explain the following concepts:
   - stock's beta
   - expectations hypothesis of interest rates
   - systematic risk
   - type B arbitrage
   - put-call parity
   - swap.

2. Are the following statements true or false? Justify your answers.
   a) Delta of a European call option may assume values only within the range [0,1].
   b) An Asian option can be priced using a binomial lattice.
   c) Sharpe's index is well suited to the measurement of the efficiency of individual stocks.
   d) If the maturity of bond A is longer than the maturity of bond B, then the duration of A is always greater than the duration of B.
   e) A stock and the risk-free asset can be used to construct a synthetic option whose delta is the same as the delta of a corresponding market-traded option.
   f) The yield of a bond is defined so that it is the same as the internal rate of return of the cash flow stream implied by the bond.

3. Let us consider the Capital Asset Pricing Model (CAPM):
   a) What assumptions is this model based on?
   b) How can CAPM be employed as a pricing tool?
   c) If all investors were to behave according to the dictates of CAPM, what kinds of portfolios would they form?
   d) What challenges are involved in the numerical estimation of the model parameters?

(turn the page, please)

4. A farmer estimates to harvest 30 tons of wheat in 4 months. He is worried about possible price changes, so he considers hedging. Currently the spot prices are 2.00 euro/kg for wheat and 1.60 euro/kg for barley. The annual standard deviation of the spot price for both wheat and barley is 18%, and the correlation between these prices is 0.7. There is no forward contract for wheat, but there is a forward contract for barley.

   What is the minimum-variance hedge for the farmer when the hedge is constructed using 4-month forward contracts for barley? How much smaller is the standard deviation of a position that is hedged in this way when compared to the situation with no hedge?

5. The spot rate for year 1 is 5%. Construct a short rate lattice for four years when risk-neutral probabilities are $q = 1-q = 50\%$ and the parameter values for the binomial lattice are $u = 1.2$ and $d = 0.9$.

   a) Using this lattice, find the spot rate for year 2.
   b) Find the value at which a public institution should issue a 2-year bond in two years from now, when the principal of the bond is 6 000 euro and the annual coupon rate is 10%.
   c) Consider a European call option which gives its holder the right to buy the aforementioned bond in two years from now at the price of 6 200 euro. What is the current price of this option?