S.72-1140 Transmission Methods in Telecommunication Systems

Closed-book Exam on Tuesday 8.1.2008

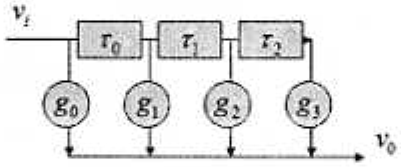
1. The average amount of information / time unit for a source emitting M kind of symbols is called as source entropy

$$H(P)=\sum_{i=1}^{M} P_i I_i = \sum_{i=1}^{M} P_i \log_2 \frac{1}{P_i} \qquad \text{bits /symbol,}$$

where the symbol probabilities P_i are defined such that $\sum P_i = 1$.

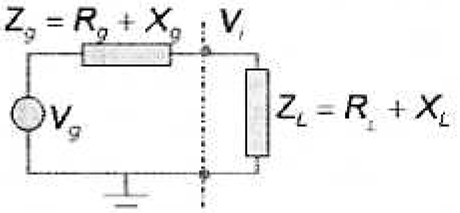
How much information is convoyed by *M* symbols having alphabet probabilities of (i) {1/2, 1/2}, (ii) {1/4, 3/4} , (iii) {1/3, 1/3, 1/3} , and (iv) {1/2, 1/4, 1/4}. Summarize your conclusions!

2. Linear channels can be described by Finite Impulse Response (FIR) – filter shown below. Determine the respective transfer function.



3. A block code consists of the following codes 10011, 11101, 01110, 00000.
a) How many errors can be detected /corrected by this code?
b) Is this a linear code?

4. (i) Determine channel input impedance Z_L for $V_g = 1\,V$, $R_g = 50\,\Omega$ and $V_i = 0.2\,V$, (ii) Assume load impedance is $Z_L = 50 + j10\,\Omega$ and $Z_g = 50\,\Omega$. How much is the dissipation angle θ?



5. For a (6,3) systematic linear block code, the three parity-check digits c_4, c_5 and c_6 are: $c_4 = d_1 + d_2 + d_3, c_5 = d_1 + d_2, c_6 = d_1 + d_3$

a) Construct the appropriate generator matrix for this code
b) Construct the code(s) generated by this matrix
c) Determine the error correction capabilities of the code
d) Prepare a suitable decoding table
e) Decode the word: 101100

Collection of Formulas

$$C = W_C \cdot \log_2(1+SNR)\ ,\ \begin{cases} r_{max} = 2B_T = r_b/n = r_b/\log_2(L) \\ \Rightarrow r_b = 2B_T\log_2(L), L = 2^n \end{cases},\ r = n\cdot f_S$$

$$C = W_C \cdot \log_2(1+SNR)$$

$$P_{dB} = 10\log(P_1/P_2), P_{dB} = 20\log(V_1/V_2), P_{dBm} = 10\log(P_1/1mW)\ ,\ \frac{V_g}{V_i} = \frac{Z_g + Z_L}{Z_L}$$

$$\begin{cases} \text{y(t)=Kx(t - t}_d) \\ \Rightarrow \text{Y(f)=}\mathcal{F}[\text{y(t)}]=\underbrace{\text{Kexp(-j}\omega\text{t}_d)}_{H(f)}X(f) \\ G_y(f) = |H(f)|^2 G_x(f)\ \text{(= output PDF)} \end{cases},\ \begin{cases} l = d_{min} - 1,\ t = \lfloor l/2 \rfloor, R_C = k/n \le 1 \\ d_{min}\big|_{max} = n - k + 1\ \text{(repetition codes)} \end{cases}$$

$$\begin{cases} N_R = \int_{-\infty}^{\infty} (\eta/2)|H_R(f)|^2 df \\ \quad = \int_{B_T} (\eta/2)df + \int_{B_T} (\eta/2)df = \eta B_T \end{cases} \begin{cases} P(n,k) = \binom{n}{k}\alpha^k(1-\alpha)^{n-k} \\ \binom{n}{k} = \frac{n!}{k!(n-k)!} \end{cases}$$

$$\begin{cases} B_T = 2|D-1|W, 1 >> D >> 1 \\ \beta = A_m f_\Delta / f_m \big|_{A_m=1, f_m=W} = f_\Delta / W \equiv D \\ B_{T,DSB} = 2W, B_{T,SSB} = W \end{cases}$$

$$\begin{cases} x_C(t) = A_C\cos(\omega_C t + \phi(t)) \\ \phi_{PM}(t) = \phi_\Delta x(t) \\ \phi_{FM}(t) = 2\pi f_\Delta \int_{t_0}^{t} x(\lambda)d\lambda, t \ge t_0 \end{cases} \quad \phi(t) = \begin{cases} \underbrace{\phi_\Delta A_m}_{\beta}\sin(\omega_m t), \text{PM} \\ \underbrace{(A_m f_\Delta / f_m)}_{\beta}\sin(\omega_m t), \text{FM} \end{cases} \quad \begin{cases} \gamma = S_R/(\eta W) \\ S_R/N_R = \gamma W/B_T \\ \gamma_b = E_b/N_0 \end{cases}$$

$$y(t) = \begin{cases} v_i(t) & \text{Synchronous detector} \\ A_v(t) - \overline{A_v} & \text{Envelope detector} \\ \phi_v(t) & \text{Phase detector} \\ d\phi_v(t)/dt & \text{Frequency detector} \end{cases},\ \begin{cases} x_{AM}(t) = A_C[1 + \mu x_m(t)]\cos(\omega_c t) \\ x_{DSB}(t) = x_m(t)\cos(\omega_c t) \end{cases}$$

$$\begin{cases} Q = R\sqrt{C/L} \\ f_0 = (2\pi\sqrt{LC})^{-1} \end{cases},\ H(\omega) = V_{out}(\omega)/V_{in}(\omega) = z_p/z_i$$

$$Q(k) = \frac{1}{\sqrt{2\pi}}\int_k^{\infty}\exp\left(-\frac{\lambda^2}{2}\right)d\lambda$$

$$\lambda = (m-x)/\sigma \Rightarrow Q(k) = \frac{1}{\sqrt{2\pi}}\int_{\sigma k+m}^{\infty}\exp\left(-\frac{(x-m)^2}{2\sigma^2}\right)dx$$

$$\begin{cases} P = UI = U^2 / R = I^2 R \\ R = U / I \end{cases}, \frac{V_g}{V_i} = \frac{Z_g + Z_L}{Z_L}, P_L = V_i I_i \cos\theta$$

$$\cos\theta = R_{tot} / Z_{tot} = R_{tot} / \sqrt{R_{tot}^{\ 2} + X_{tot}^2}, X_{tot} = X_g + X_L, R_{tot} = R_L + R_g$$

$$N_{D(PM)} = \int_{-W}^{W} \frac{\eta}{2S_R} df = \frac{\eta W}{S_R},\ N_{D(FM)} = \int_{-W}^{W} \frac{\eta f^2}{2S_R} df = \frac{\eta W^3}{3S_R}$$

$$S_D / N_D|_{FM} = \frac{f_\Delta^2 S_x}{\eta W^3 / (3S_R)} = 3\underbrace{\left(\frac{f_\Delta}{W}\right)}_{D}^2 S_x \underbrace{\frac{S_R}{\eta W}}_{\gamma} = 3D^2 S_x \gamma,\ S_D / N_D|_{FM, D>>1} = \frac{3}{4}\left(\frac{B_T}{W}\right)^2 S_x \gamma$$

$$S_D / N_D|_{PM} = \frac{\phi_\Delta^2 S_x}{\eta W / S_R} = \phi_\Delta^2 S_x \gamma, \text{where } \phi_\Delta^2 S_x \le \pi^2$$

$$\begin{cases} \int \frac{1}{1+x^2} dx = \arctan(x) \\ \int \frac{x^2}{1+x^2} dx = x - \arctan(x) \end{cases} \quad \begin{cases} \prod\left(\frac{t}{\tau}\right) \leftrightarrow \tau \operatorname{sinc} f\tau \\ \Lambda\left(\frac{t}{\tau}\right) \leftrightarrow \tau \operatorname{sinc}^2 f\tau \end{cases}$$

$$\begin{cases} \frac{d^n v(t)}{dt^n} \leftrightarrow (j2\pi f)^n V(f) \\ \int_{-\infty}^{t} v(\lambda) d\lambda \leftrightarrow \frac{1}{j2\pi f} V(f) + \frac{1}{2} V(0)\delta(f) \end{cases}$$

$$\begin{cases} \sin\alpha \sin\beta = 1/2\cos(\alpha - \beta) - 1/2\cos(\alpha + \beta) \\ \cos\alpha \cos\beta = 1/2\cos(\alpha - \beta) + 1/2\cos(\alpha + \beta), \\ \sin\alpha \cos\beta = 1/2\sin(\alpha - \beta) + 1/2\sin(\alpha + \beta) \end{cases} \begin{cases} \cos^2\alpha = (1 + \cos 2\alpha)/2 \\ \cos^3\alpha = (3\cos\alpha + \cos 3\alpha)/4 \end{cases} \begin{cases} (\alpha + \beta)^2 = \alpha^2 + 2\alpha\beta + \beta^2 \\ (\alpha + \beta)^3 = \alpha^3 + 3\alpha^2\beta + 3\alpha\beta^2 + \beta^3 \end{cases}$$