

Mat-1.1030 Basic course in mathematics L3

partial exam 3 14.12.2009

Please fill in clearly *on every sheet* the data on you and the examination. On *Examination code* mark course code, title and text mid-term or final examination. Degree Programmes are ARK, AUT, BIO, EST, ENE, GMA, INF, KEM, KTA, KON, MAR, MTE, PUU, RRT, TFM, TIK, TLT, TUO, YYT.

No calculators allowed.

1. For which values of a the system $\mathbf{x}'(t) = \mathbf{A} \mathbf{x}(t)$ is stable or asymptotically stable, when

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & a-1 \\ 0 & a+1 & -a \end{bmatrix} .$$

2. Consider problem

$$x'(t) = -x(t) + \alpha t x^2(t) , \quad x(0) = e ,$$

and interpret the solution also as a function of α . Compute $\frac{\partial}{\partial \alpha} x(t, \alpha)|_{t=1, \alpha=0}$.

3. Find the equilibria of the system

$$\begin{cases} x_1' = \sin(\pi x_2) \\ x_2' = x_1 - x_2 \end{cases}$$

and linearize the system around these. Classify the linear systems. Can the stability of the nonlinear system around the equilibria be deduced from the linearizations?

4. Consider the following implicit method

$$\mathbf{x}_{j+1} = \mathbf{x}_j + \frac{h}{2} (\mathbf{f}(t_j, \mathbf{x}_j) + \mathbf{f}(t_j + h, \mathbf{x}_{j+1})) .$$

- Write this in the usual Runge-Kutta table form.
- What is the order of the method when applied to the problem $x' = \lambda x$?
- Find the stability region of the method.