Exam: Mathematical geodesy 24.05.2013

Function calculator

Formula collection:

Let the sides of a spherical triangle ABC be a, b, c and the opposite angles of the sides α, β, γ. Then:

$$\cos c = \cos a \cos b + \sin a \sin b \cos \gamma$$

$$\frac{\sin a}{\sin \alpha} = \frac{\sin b}{\sin \beta} = \frac{\sin c}{\sin \gamma}$$

Oblateness and eccentricity (a, b the long and short semi-axes of the Earth ellipsoid):

$$f = \frac{a-b}{a},\ e^2 = \frac{a^2 - b^2}{a^2}.$$

Latitudes: geographical φ, geocentric ϕ, reduced β:

$$\tan \varphi = \frac{a^2}{b^2} \tan \phi = \frac{a}{b} \tan \beta.$$

1. **Co-ordinates and transformations**

   Show that the rotation matrix

   $$\begin{bmatrix} \cos\kappa \cos\omega & -\cos\kappa \sin\omega & -\sin\kappa \\ \sin\kappa \cos\omega & -\sin\kappa \sin\omega & \cos\kappa \\ \sin\omega & \cos\omega & 0 \end{bmatrix}$$

   is *orthogonal*.

2. **Spherical trigonometry**

   Compute the distance between Helsinki ($\varphi = 60°, \lambda = 25°$) and Copenhagen ($\varphi = 56°, \lambda = 12°$) in angular units ($°$). Spherical geometry.

3. **The reference ellipsoid**

   Given $a = 6378388\,\text{m}$, $f = 1/297$ (International ellipsoid).

   (a) Compute e^2 and b.

   (b) If $\varphi = 45°$, compute ϕ.

4. **Surface theory**

   Given the surface of a hill in the form of a semi-sphere, height H, and its parametrisation (h, A):

   $$\mathbf{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \sqrt{H^2 - h^2} \cos A \\ \sqrt{H^2 - h^2} \sin A \\ h \end{bmatrix}.$$

   Compute the pair of tangent vectors

   $$\mathbf{x}_h = \frac{\partial \mathbf{x}}{\partial h},\ \mathbf{x}_A = \frac{\partial \mathbf{x}}{\partial A}$$

   and the metric tensor

   $$g_{ij} = \begin{bmatrix} g_{hh} & g_{Ah} \\ g_{hA} & g_{AA} \end{bmatrix} = \begin{bmatrix} \mathbf{x}_h \cdot \mathbf{x}_h & \mathbf{x}_A \cdot \mathbf{x}_h \\ \mathbf{x}_h \cdot \mathbf{x}_A & \mathbf{x}_A \cdot \mathbf{x}_A \end{bmatrix}.$$

5. **Map projections**

The Mercator projection formulas are on the surface of the sphere

$$\begin{aligned} x &= \lambda, \\ y &= \ln \tan\left(\frac{\pi}{4} + \frac{\varphi}{2}\right), \end{aligned}$$

where (φ, λ) are geographical co-ordinates and (x, y) map co-ordinates.

A distance element of the Earth surface is $dS^2 = R^2 d\varphi^2 + R^2 \cos^2 \varphi \, d\lambda^2$ and in the map plane $ds^2 = dx^2 + dy^2$.

Compute the scale factors m_1 and m_2 in the West-East and South-North directions.

:

	1	2	3 a b	4	5	Yht.
	5	5	5 2 3	5	5	25

	10	12	16	19	23
	1	2	3	4	5