Helsinki University of Technology
Laboratory for Theoretical Computer Science
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T-79.1002 Introduction to Theoretical Computer Science Y (2 cr)
Exam Tue 12 May 2009, 4 a.m. – 7 p.m.

Write down on each answer sheet:
- Your name, department, and student number
- The text: "T-79.1002 Introduction to Theoretical Computer Science Y 12.5.2009"
- The total number of answer sheets you are submitting for grading

1. Finite state automata.

   (a) Let

   $$L = \{w \mid w = a_1 a_2 \ldots a_n, n \geq 0, a_i \in \{0,1\}, a_1 = a_3 = a_5 = \ldots = 1\}.$$

   In other words, L consists of those binary strings that have 1s in odd-numbered positions. Describe L with a finite state automaton. *5 p.*

   (b) Let

   $$L = \{w \mid w \in \{0,1\}^*, w \text{ contains an even number of 0s or exactly two 1s.}\}.$$

   Describe L with a finite state automaton. *5 p.*

2. Regular expressions.

   (a) Let

   $$L = \{w \mid w = a_1 a_2 \ldots a_n, n \geq 0, a_i \in \{0,1\}, a_1 = a_3 = a_5 = \ldots = 1\}.$$

   In other words, L consists of those binary strings that have 1s in odd-numbered positions. Describe L with a regular expression. *5 p.*

   (b) Let

   $$L = \{w \mid w \in \{0,1\}^*, w \text{ contains an even number of 0s or exactly two 1s.}\}.$$

   Describe L with a regular expression. *5 p.*

3. A context-free language.

   Let us examine the language $L = \{a^m b^n \mid m < n\}$.

   (a) Design a context-free grammar that generates the language L. *5 p.*

   (b) Give the leftmost derivation and parse tree for the string *aabbb*. *5 p.*

4. Regular languages

   Let L be a regular alphabet over the alphabet Σ. Let $\Gamma \subseteq \Sigma$. Show that $L \cap \Gamma^*$ is regular.

   (In other words, show that a regular language remains regular, if we remove some characters from its alphabet and all strings that contain one or more of the removed characters.) *10 p.*

Total 40 p.