Helsinki University of Technology
Department of Communications and Networking

EXAM
18.12.2009

S-72.2410 Information Theory

1. (1p.) Will you give course feedback at `http://palaute.ee.hut.fi` no later than on December 21, 2009 = NEXT MONDAY (or have you already done so)? (Yes = 1p. No = 0p.)

2. (8p.) Entropy. (Exercise 1, problem 1.)

   Let $p(x)$ be given by

$X\backslash Y$	0	1
0	1/3	1/2
1	1/6	0

   (a) Find $H(X)$,
   (b) find $H(Y)$,
   (c) find $H(X \mid Y)$,
   (d) find $H(Y \mid X)$,
   (e) find $H(X,Y)$,
   (f) find $H(Y) - H(Y \mid X)$,
   (g) find $I(X;Y)$, and
   (h) draw a Venn diagram for the quantities in (a) through (g).

3. (8p.) Source coding.

   Consider the random variable

   $$X = \begin{pmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 \\ 0.49 & 0.26 & 0.12 & 0.04 & 0.04 & 0.03 & 0.02 \end{pmatrix}.$$

   (a) (3 p.) Find a binary Huffman code for X.
   (b) (3 p.) Find a binary Shannon code for X.
   (c) (2 p.) Find the expected codelength for both (a) and (b). Comment.

4. (8p.) Channel capacity.

   Consider a multiple access channel with three senders X_1, X_2, X_3 such that $X_i \in \{0,1\}$ and

   $$Y = \begin{cases} 1, & \text{if } X_1 + X_2 + X_3 \geq 2, \\ 0, & \text{otherwise.} \end{cases}$$

   (a) (3 p.) What is the capacity region of the multiple access channel?
   (b) (2 p.) Describe a method for channel coding that allows any combination of rates in the capacity region.
   (c) (3 p.) Suppose that we have a situation where X_1 and X_2 are sending independent bits with uniform distribution. Then X_1 and X_2 can be viewed as noise from the viewpoint of X_3. What is then the capacity of the channel observed by the third sender, $X_3 \rightarrow Y$?