

4. (8p.) Source coding. The weather on consecutive days in Reykjavik is a Markov chain and given by the probability transition matrix in Figure 1. There are three weather conditions, $\{C, R, S\}$, cloud, rain and sunshine. For example, if it is cloudy (C) today, then the sun shines (S) tomorrow with probability 0.1.
- Determine the stationary distribution for the weather condition.
 - What is the entropy rate for the weather condition, assuming that the Markov chain is stationary.
 - Design a (good) binary source code for the weather in the following way: Construct three different codes for the weather condition tomorrow, given the weather condition today (these should be optimal in each separate case). That is, one code when it is Cloudy today, one when it is Rainy, and one when it is Sunny. So at each step of the encoding, we first need to decide which code to use next. This idea gives a uniquely decodable code for any Markov chain. Use your coding scheme to encode SSSRRSRCCC (the code for Cloudy weather is used in the first step).
 - What is the expected amount of bits needed per day for the coding scheme designed in the previous part. Compare this with the entropy rate. Briefly explain (without calculations) how one could improve the coding scheme to get closer to the entropy rate.


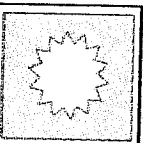
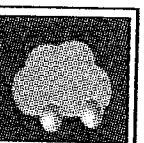

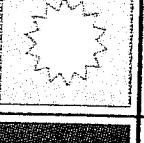

			
	0,8	0,1	0,1
	0,2	0,7	0,1
	0,1	0,3	0,6

Figure 1: Probability transition matrix