[redacted]

1. Let $f : \mathbb{C} \to \mathbb{C}$ be the mapping defined by condition

$$z = x + \mathrm{i}y \mapsto x^3 + \mathrm{i}(1-y)^3.$$

Find those points z in the complex plane, where f is

(a) differentiable,

(b) analytic.

(c) Find the derivative $f'(z) \in \mathbb{C}$ of f at those points where it exists.

2. Find a Möbius mapping, that maps domain $\{z \in \mathbb{C} |\ |z-1| < 1\}$ to domain $\{z \in \mathbb{C} |\ \operatorname{Re} z > 1\}$.

3. Find the value of function

$$f : f(z) = \oint_\gamma \frac{2w^2 - w - 1}{w(w-z)e^{\pi \mathrm{i}(w-1)}} \mathrm{d}w$$

at points

(a) $z = 1$

(b) $z = 2$

(c) $z = 4$

when integration is performed along path γ that parametrizes circle

$$S(0,3) = \{z \in \mathbb{C} |\ |z| = 3\}$$

once counterclockwise.

4. Find two series in terms of powers of variable z for function f defined by conditon

$$z \mapsto \frac{1}{z(1+z^2)}.$$

Where are these series valid?