Helsinki University of Technology, Department of Information and Computer Science TJ

T-79.3001 Logic in Computer Science: Foundations

Examination, May 7, 2009

Please note the following: your answers will be graded only if you have passed all the three home assignments before the exam!

Assignment 1 (10*p*)

(a) Define the following concepts: *modus ponens*, *disagreement set*, and *free variable occurrence*. ($3 \times 2p$)

(b) What is meant by the notation $\Sigma \models \phi$?

Prove in detail that if $\Sigma \models \phi$ and $\Sigma \models \neg\phi$ for some sentence ϕ, then Σ is unsatisfiable. (4*p*)

Assignment 2 (10*p*) Prove the following claims using semantic tableaux:

(a) $\models (A \rightarrow B) \land (\neg A \rightarrow \neg B) \rightarrow (A \land \neg B \leftrightarrow B \land \neg A)$.

(b) $\models \forall x(\exists y R(x,y) \rightarrow P(x)) \rightarrow \forall y \forall x(P(x) \lor \neg R(x,y))$.

Tableau proofs must contain all intermediary steps !!!

Assignment 3 (10*p*) Derive a Prenex normal form and a clausal form (i.e. a set of clauses S) for the sentence

$$\neg(\exists x(P(x) \lor \forall y Q(x,y)) \rightarrow \exists y(P(y) \lor Q(y,y))).$$

Make S as simple as possible. Prove that S is unsatisfiable using resolution.

Assignment 4 (10*p*) Let us represent strings "", "*a*", "*b*", "*aa*", "*ab*", "ba", "bb", ... that consist of letters a ja b using ground terms

$$e,\ a(e),\ b(e),\ a(a(e)),\ a(b(e)),\ b(a(e)),\ b(b(e)),\ \ldots,$$

built of a constant symbol e, which represents the empty string "", and unary functions $a(x)$ and $b(x)$, that append the respective letter a or b at the beginning of a string x. Thus $a(b(e))$ is interpreted as $a(b(\text{""})) = a(\text{"}b\text{"}) = \text{"}ab\text{"}$.

(a) Define predicate $U(x)$ = "the string x has either occurrences of the letter a only or occurrences of the letter b only but not both" using predicate logic so that your definition covers all finite strings as described above.

(b) Give a model $\mathcal{S} \models \Sigma$ of your definition Σ on the basis of which it holds that

$$\Sigma \not\models U(e).$$

Assignment 5 (10*p*)

Explain how the *weakest precondition* B_1 of an if-statement

$$\texttt{if}\,(B)\ \texttt{then}\ \{C_1\}\ \texttt{else}\ \{C_2\}$$

can be formed given a postcondition B_2 for it.

Consider the following program `Double`:

```
v=0; z=x; while(!(z==0)) {z=z-1; v=v+2}.
```

Use weakest preconditions and a suitable invariant to establish

$$\models_p [\texttt{true}]\ \texttt{Double}\ [\texttt{v==2*x}].$$

The name of the course, the course code, the date, your name, your student number, and your signature must appear on every sheet of your answers.

Feedback: `http://www.cs.hut.fi/u/vjs/php/palaute.php?year=2009&term=k`