S-72.2410 Information Theory

1. (1p.) Do you intend to give feedback at http://palaute.ee.hut.fi no later than on December 19, 2008 = TODAY (or have you already done so)? (Yes = 1p. No = 0p.)

2. (8p.) Entropy. Consider two random binary variables, X and Y, with joint distribution $p(x,y)$ fulfilling $p(0,0) = 0.48$, $p(0,1) = 0.32$, $p(1,0) = 0.12$, $p(1,1) = 0.08$. Moreover, let Z be a ternary variable fulfilling $Z = X + Y$.

   (a) Determine $H(X)$ and $H(Y)$.

   (b) Are X and Y independent or not? Motivate your answer.

   (c) Determine $H(Z|X)$.

   (d) Determine $I(Z;Y)$.

3. (8p.) Channel capacity. Motivate your answers, just answering yes or no or giving one of the alternatives is not sufficient. The motivation should be concise though.

   (a) If you wish to increase the capacity of a noisy continuous time communication channel with bandwidth 100 Hz and signal-to-noise ratio 41 dB, is it more effective to increase its bandwidth or to improve its signal-to-noise ratio?

   (b) Assume that you want to transmit data (pictures) from Pluto at a rate that is 1000 times higher than the rate of current space probes. What is the core issue to achieve this? Unlimited bandwidth is assumed.

   (c) If there are three binary symmetric channels available with crossover probabilities $p = 0.40$, $p = 0.50$, and $p = 0.70$, respectively, which one would you choose to use?

   (d) Consider three independent Gaussian channels in parallel,

$$Y_j = X_j + Z_j, \quad Z_j \sim \mathcal{N}(0, N_j),$$

   with $N_1 = 1.5$ watts, $N_2 = 4$ watts, and $N_3 = 2.5$ watts. How should one distribute a power of 3 watts among the three senders to achieve maximum total capacity. **Hint:** water-filling.