Mat-1.1631 Mathematics 3-I

Please fill in the required information into each paper sheet.
Calculators are allowed.

1. (a) Evaluate $\left(\dfrac{1-i}{1+i}\right)^8$. (2p)

(b) Sketch the set $|z\text{-}i|=|z\text{-}1|$ in the complex plane. (2p)

(c) Find all zeros of the function $f(z)=\sin z$. (2p)

2. (a) State the residue theorem (the residue integration method). (1p)

(b) Evaluate $\displaystyle\oint_{|z-3|=4}\frac{15z+9}{z^3-9z}dz$, integration is counterclockwise. (5p)

3. (a) Does the function $y=\exp(x^2)$ have a Laplace transform? (1p)

(b) Solve the initial value problems by the Laplace transform: $y'-y=1$, $y(0)=-1$. (5p)

4. (a) Can a discontinuous function be developed in a Fourier series? (Give a reason for your answer) (2p)

(b) Does the Fourier sine series of the function $f(x)=\cos^2 x$ exist? (1p)

(c) Find the Fourier series of $f(x)=\cos^2 x$ on $[-\pi, \pi]$. (2p)

(d) State the convergence theorem for Fourier series. (1p)

5. Find the Fourier sine transform of the function $f(x)=\begin{cases}x^2 & \text{if } 0<x\le 1\\ 0 & x>1\end{cases}$. (6p)

Appendix A. Fourier series

Any periodic (with period 2L) piecewise continuous in the interval $-L\le x\le L$ function $f(x)$ can be represented by the Fourier series

$$f(x)=a_0+\sum_{n=1}^{\infty}\left(a_n\cos\frac{\pi n x}{L}+b_n\sin\frac{\pi n x}{L}\right),$$

where $\displaystyle a_0=\frac{1}{2L}\int_{-L}^{L}f(x)\,dx \qquad a_n=\frac{1}{L}\int_{-L}^{L}f(x)\cos\frac{\pi n x}{L}dx$, $n=1,2\ldots$

$\displaystyle b_n=\frac{1}{L}\int_{-L}^{L}f(x)\sin\frac{\pi n x}{L}dx$, $n=1,2\ldots$