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1. Explain briefly, with 20–40 words or a mathematical definition, the following concepts or abbreviations: 6p.

- (i) Mach bands
- (ii) discrete Laplace operator
- (iii) salt and pepper noise
- (iv) linear position-invariant degradation process
- (v) morphological hit-or-miss transformation
- (vi) chromaticity diagram

2. Different types of distance measures will be studied in two-dimensional discrete space. (i) What conditions must a general function  $D$  fulfill in order to be a distance function or metric? (ii) Give the definitions of the Euclidan distance  $D_e(p, q)$ , the city-block distance  $D_4(p, q)$ , and the chessboard distance  $D_8(p, q)$ . (iii) Let two points be  $p = (2, 5)$  and  $q = (8, 9)$ . Examine and draw which points  $r$  of the discrete plane  $(x, y) \in [0, 15] \times [0, 15]$  fulfill the condition  $D_e(p, r) = D_e(q, r)$ . (iv) Examine similarly points  $s$  with the condition  $D_4(p, s) = D_4(q, s)$  and points  $t$  with the condition  $D_8(p, t) = D_8(q, t)$ . (v) Analyze your findings. 6p.

3. We will study the use of wavelets in multiresolution processing of a  $4 \times 4$  -sized image. (i) Haar's scaling function is

$$\varphi(x) = \begin{cases} 1 & , 0 \leq x < 1 \\ 0 & , x < 0 \vee x \geq 1 \end{cases}$$

Show graphically the set of expansion functions  $\varphi_{j,k}(x) = 2^{j/2}\varphi(2^j x - k)$ , produced from the above function, that is needed when the length of the processed sequence is four. (ii) Show mathematically how does Haar's wavelet function  $\psi(x)$  look like and draw the wavelet set that corresponds to the above expansion functions. (iii) Show how  $\varphi_{1,0}(x)$  and  $\varphi_{1,1}(x)$  can be formed by using  $\varphi_{0,0}(x)$  and  $\psi_{0,0}(x)$ . What is the more general dependence of which this is an example? (iv) Form from  $\varphi_{1,0}(x)$ ,  $\varphi_{1,1}(x)$ ,  $\psi_{1,0}(x)$  and  $\psi_{1,1}(x)$  the transform matrix  $\mathbf{H}$  that can be used in multiresolution processing of a  $4 \times 4$  -sized image. (v) By using the above matrix, calculate the transform  $\mathbf{T} = \mathbf{H}\mathbf{F}\mathbf{H}^T$ , where the analyzed image is

$$\mathbf{F} = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(vi) Comment the operations you performed, study the contents of matrix  $\mathbf{T}$  in view of multiresolution processing and explain what has been the sense in all this. 6p.