Mat-5.156 Theory of Elasticity **Stenberg**

Examination 29.10.2009

1. A linearly elastic body Ω is loaded by a body load $\boldsymbol{f}$. Write down the total energy, and derive the corresponding variational problem. Starting from the variational problem, write down the equilibrium equations for linear elasticity. *Hint: you can assume the stress tensor to be symmetric without proving it yourself.*

2. Compute the pricipal stresses and the corresponding pricipal directions for the following pure shear stress state

$$\boldsymbol{\sigma} = \begin{pmatrix} 0 & \sigma_{12} & 0 \\ \sigma_{12} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

3. Write down the boundary value problem and draw a picture for the problem of minimizing the functional

$$J(v) = \frac{1}{2}\int_0^L EI(v''(x))^2\,dx - Gv(L/2) + kv(L)^2$$

in the space

$$K = \{\, v \mid \int_0^L EI(v''(x))^2\,dx < \infty,\, v(0) = 0 \,\}.$$

4. We say that an elastic body is in a strainless state if the small strain tensor vanish identically. Show that this is equivalent with the condition that the displacement is an infinitesimal rigid body motion.