HELSINKI UNIVERSITY OF TECHNOLOGY
Signal Processing Laboratory
S-88.212 Signal Processing in Telecommunications II (2 cu)

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Write in each answer paper your name, department, student number, the course name and code, and the date. Number each paper you submit and denote the total no. of pages. 5 problems, 30 points total. Papers in English only. Please feel free to answer in English, Finnish or Swedish. The BETA mathematical tables can be utilized – you can borrow a copy from the exam supervisor if you do not have your own.

1. (1p each) Define and describe *briefly* (2..3 lines of text) the following concepts:
   a) Farrow structure
   b) Costas loop
   c) TED
   d) Cramer-Rao bound
   e) Synchronous sampling
   f) NCO

3

2. (6p) Assume that we wish to estimate the amplitude A of a sinusoid embedded in white Gaussian noise (WGN) $w(k)$:

$$r(k) = A\cos(2\pi f_0 k + \theta) + w(k), \qquad k = 0,1,\ldots, N-1 \tag{1}$$

where $r(k)$ is the discrete-time received signal sequence of N samples. The frequency f_0 and phase θ are assumed known. The probability density function (PDF) is

6

$$p(\mathbf{r}; A) = \frac{1}{\left(2\pi\sigma^2\right)^{\frac{N}{2}}} \exp\left\{-\frac{1}{2\sigma^2}\sum_{k=0}^{N-1}\left[r[k] - A\cos(2\pi f_0 k + \theta)\right]^2\right\} \tag{2}$$

where σ^2 is the noise variance. Find a Maximum Likelihood (ML) estimator for the amplitude A in closed form. (Hint: find the log-likelihood function first!)

3. (6p) Describe and define the DA, DD, and NDA estimation strategies for synchronization parameters in digital passband communications systems. Discuss the advantages and disadvantages of different schemes.

4. (6p) For clockless NDA phase recovery, we have derived the phase estimator

$$\theta = \frac{1}{M}\arg\left\{\sum_{k=0}^{NL_0-1} x^M(kT/N)\right\} \tag{3}$$

where N is the oversampling factor and M is the number of MPSK constellation points. Assuming raised-cosine pulses with excess bandwidth factor α, derive the expression for the minimum oversampling factor N. $F_m = \frac{(1+\alpha)\pi}{T}$ $N=$

5. (8p – special bonus problem!) For channel estimation, the system model for the received signal vector is

$$\mathbf{r} = \mathbf{A}\mathbf{h} + \mathbf{w} \tag{4}$$

with the matrix of known data symbols

$$\mathbf{A} = \begin{bmatrix} \mathbf{a}_1^{\mathrm{T}} \\ \mathbf{a}_2^{\mathrm{T}} \\ \vdots \\ \mathbf{a}_L^{\mathrm{T}} \end{bmatrix} = \begin{bmatrix} a_N & a_{N-1} & \cdots & a_1 \\ a_{N+1} & a_N & \cdots & a_2 \\ \vdots & \vdots & \ddots & \vdots \\ a_{N+L-1} & a_{N+L-2} & \cdots & a_L \end{bmatrix} \tag{5}$$

and received signal vector, channel coefficient vector, and noise vector

$$\mathbf{r} = [r(1) \quad r(2) \quad \cdots \quad r(L)]^{\mathrm{T}}, \mathbf{h} = [h(1) \quad h(2) \quad \cdots \quad h(N)]^{\mathrm{T}},$$
$$\mathbf{w} = [w(1) \quad w(2) \quad \cdots \quad w(L)]^{\mathrm{T}} \tag{6}$$

The probability density function (PDF) of the received signal vector is now

$$p(\mathbf{r};\mathbf{h}) = \frac{1}{(2\pi)^{L/2}(\det(\mathbf{C}_w))^{1/2}} \exp\left[\frac{-1}{2}\left((\underbrace{\mathbf{r} - \mathbf{A}\mathbf{h}}_{w})^{\mathrm{H}}\mathbf{C}_w^{-1}(\mathbf{r} - \mathbf{A}\mathbf{h})\right)\right] \tag{7}$$

where $\mathbf{C}_w$ is the noise covariance matrix. The ML channel estimate is known to be

$$\mathbf{h}_{ML} = (\mathbf{A}^{\mathrm{H}}\mathbf{C}_w^{-1}\mathbf{A})^{-1}\mathbf{A}^{\mathrm{H}}\mathbf{C}_w^{-1}\mathbf{r} \tag{8}$$

a) (4p) Derive the ML estimate (8) from (4)-(7), carefully explaining what is done and why. Modify the solution for the AWGN case. N ei ole korrelaatiota C_w^{-1} pois

We have sent the symbols $[a_1, a_2, \ldots, a_4]$= [-1, 1, 1, -1] and received the samples $[r(1), r(2), \ldots, r(3)]$ = [-3.1, 2.1, 3.9], sampled at the symbol rate. Let us assume a linear channel with additive white Gaussian noise.

b) (4p) Determine a 1-tap FIR ML channel estimate

COURSE FEEDBACK BONUS (1p)

You will gain 1p bonus (directly in the exam points, not scaled homework points!) for a completed feedback form. For bookkeeping, please notify the exam supervisor when returning your papers.