HELSINKI UNIVERSITY OF TECHNOLOGY **Exam 12 Dec. 2002**
Signal Processing Laboratory Timo I. Laakso
S-88.212 Signal Processing in Telecommunications II (2 cu)
S-88.J Postgraduate version (2 cu, or 4 cu with seminar presentation)

Write in each answer paper your name, department, student number, the course name and code, and the date. Please tell if you want to pass the course as a postgraduate version. Number each paper you submit and denote the total no. of pages. 5 problems, 30 points total. Papers in English only. Please feel free to answer in English, Finnish or Swedish.

1. (1p each) Define and describe *briefly* (2..3 lines of text) the following concepts:
   a) Synchronous sampling
   b) GPS
   c) NDA phase estimation
   d) S-curve
   e) Fractional delay filter
   f) VCO

2. (6p) Assume that we wish to estimate the amplitude A of a sinusoid embedded in white Gaussian noise (WGN) $w(k)$:

$$r(k) = A\cos(2\pi f_0 k + \theta) + w(k), \qquad k = 0,1,\ldots, N-1 \qquad (1)$$

where $r(k)$ is the discrete-time received signal sequence of N samples. The frequency f_0 and phase θ are assumed known. The probability density function (PDF) is

$$p(\mathbf{r}; A) = \frac{1}{(2\pi\sigma^2)^{\frac{N}{2}}} \exp\left\{-\frac{1}{2\sigma^2}\sum_{k=0}^{N-1}[r[k] - A\cos(2\pi f_0 k + \theta)]^2\right\} \qquad (2)$$

where σ^2 is the noise variance. Find the Cramer-Rao bound for the estimator for A in closed form. (Hint: the Cramer-Rao bound for a scalar parameter is defined as the negative inverse of the second derivative of the PDF with respect to the parameter in question.)

3. Consider the carrier recovery problem in a 4-PSK system. Assume that after the first (analog) carrier demodulation stage a low-frequency modulating sinusoid still remains so that the appropriate baseband signal model is

$$r(t) = e^{j(2\pi f t + \theta)} \sum_i a_i h_T(t - iT - \tau) + w(t) \qquad (3)$$

where f is the carrier frequency (error) to be estimated, carrier phase θ is unknown but the timing error τ is known. Data symbols a_k are known because a training signal is used at the beginning of transmission.

(a) (3p) Derive the matched filter output $z(k) = a_k^* y(k) = e^{j[2\pi f(kT+\tau)+\theta]} + n'(k)$ that can be developed under certain assumptions (Nyquist pulse, small enough f), as explained in the lecture (slides).
(b) (3p) You have two samples of $z(k)$ available. Develop an estimator for the carrier frequency.

4. (9p TOTAL - SPECIAL BONUS PROBLEM!) An Nth-order allpass filter has a transfer function of the form

$$H(z)=\frac{z^{-N}A(z^{-1})}{A(z)}=\frac{a_N+\ldots+a_1 z^{-N+1}+z^{-N}}{1+a_1 z^{-1}+\ldots+a_N z^{-N}} \tag{4}$$

a) (4p) Design a *first-order allpass interpolator* such that its group delay equals to $(1+\mu)$ at the zero frequency. Check also that your filter is stable!

Hint: the group delay of the allpass filter is determined by the denominator group delay so that

$$\tau_{g,H}(\omega)=N+2\tau_{g,A}(\omega)$$
$$\tau_{g,A}(\omega)=\frac{-d\theta_A(\omega)}{d\omega},\quad \theta_A(\omega)=\arg\left\{A(e^{j\omega})\right\} \tag{5}$$

(Note: the signs may be defined slightly differently from what you have seen before – be careful!)

b) (3p) Determine a 2-tap Lagrange interpolator such that its group delay equals to μ at the zero frequency.

c) (2p) Compare your allpass and Lagrange interpolators and discuss their properties and suitability for symbol timing applications.

5. (6p) Channel estimation (CE) principles: explain the difference between channel equalization and CE. What is blind CE?