Write in each answer paper your name, department, student number, the course name and code, and the date. Number each paper you submit and denote the total no. of pages. 5 problems, 30 points total. Papers in English only. The BETA mathematical tables can be utilized – you can borrow a copy from the exam supervisor if you do not have your own. A basic calculator can be used (no memory, no graphics).

The homework bonus will be valid for possible future exams too.

1. (1p each) Define and describe *briefly* (2..3 lines of text) the following concepts:

a) Echo cancellation
b) Matched filter
c) Nyquist criterion
d) Channel capacity
e) OFDM
f) ZF equalizer

2. Nonlinear equalizers (6p):

Describe and compare the DFE equalizer and the Viterbi algorithm (the latter provided with a simple channel estimator). Discuss the advantages and disadvantages applications like in mobile communications channels (with fading) and wireline channel.

3. Adaptive filters (6p):

Let us consider a discrete-time model for a communication system in a linear channel (sampled at the symbol rate). The received signal samples $r(k)$ are filtered by an N-tap FIR filter (equalizer). The equalizer output can be expressed as

$$y(k) = \mathbf{h}_R^T \mathbf{r}(k). \tag{1}$$

where $\mathbf{h}_R$ and $\mathbf{r}$ are N-dimensional column vectors. The mean squared error (MSE) can be expressed as

$$E\left[e^2(k)\right] = E\left[a_k^2\right] - 2\mathbf{h}_R^T\mathbf{p}_0 + \mathbf{h}_R^T\mathbf{R}\mathbf{h}_R \tag{2}$$

where $\mathbf{p} = E\left[\mathbf{r}(k)a_k\right]$ and $\mathbf{R} = E\left[\mathbf{r}(k)\mathbf{r}^T(k)\right]$.

a) (2p) Derive the optimal minimum-MSE equalizer.
b) (2p) The general structure for an adaptive equalizer using *MSE gradient* (MSEG) algorithm is of the form

$$\mathbf{h}_R[j+1] = \mathbf{h}_R[j] - \frac{\beta}{2}\nabla_{h_R} \mathrm{E}\left[e^2(k)\right]. \tag{3}$$

Derive the MSEG algorithm for the equalizer.
c) (2p) Consider a 1-tap adaptive MSEG filter. Derive the criterion for the adaptation convergence in terms of the step parameter β.

4. Nyquist and Matched filtering criteria (9p bonus):

Let us derive pulse waveforms which meet the Nyquist criterion:

a) (3p) Assume an ideal baseband communication system of data rate $1/T$ in an AWGN channel which needs no excess bandwidth. Define the *pulse spectrum* and derive the *continuous-time pulse waveform* via inverse Fourier transform. No filtering assumed in the receiver, just symbol-rate sampling.

b) (3p) The same as a) except now we assume excess bandwidth α ($0< \alpha <1$) and that the spectrum is *piecewise constant.*

c) 3p) The same as b) except now we want to design transmit and receive filters that form a matched-filter pair and whose *convolution meets the Nyquist criterion.* Assume again that the *spectrum of the convolution is piecewise constant.*

5. Channel capacity (6p):

Consider the transmission of signal $x(t)$ over a linear channel with associated impulse response $c(t)$ and frequency response $C(f)$. The output waveform of the channel is then $r(t) = c(t)*x(t)$, where '*' denotes convolution. The output of the channel is thereafter corrupted by colored noise $n(t)$ with power spectral density (PSD) $S_n(f)$.

a) (4p) Solve for the optimum transmit power spectrum $S_x(f)$ that maximizes the channel capacity when the total transmit power P_x is limited to 12 W (see Equation (5) on Page 3). To help you solve the problem, Figure 1 below provides you with the PSD of the noise $S_n(f)$ and the magnitude squared of the channel transfer function. Assume that $S_n(f) = \infty$, for $|f| > 4$ Hz.

b) (2p) Determine the channel capacity resulting from the optimized transmit power spectrum in a).

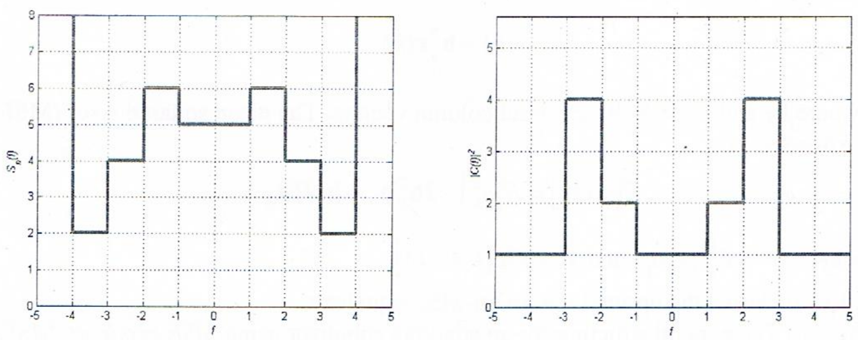

Figure 1: PSD of the noise $S_n(f)$ in W/Hz and channel gain $|C(f)|^2$ versus frequency f in Hz.

Hint: The optimal power spectrum is obtained with the water-pouring theorem as:

$$S_{x,opt}(f) = L - S_n(f)/|C(f)|^2 \tag{4}$$

whenever resulting $S_{x,\text{opt}}(f)$ is positive (zero otherwise) and the water-filling L is determined so that the total transmit power is limited, i.e.,

$$P_x = \int_{-\infty}^{\infty} S_{x,opt}(f)df \tag{5}$$

The optimal capacity is then obtained by (double-sided) integration:

$$C_{CH} = \frac{1}{2}\int_{-\infty}^{\infty} \log_2\left(1+\frac{S_{x,opt}(f)|C(f)|^2}{S_n(f)}\right)df \tag{6}$$