T-61.5140 Machine Learning: Advanced Probabilistic Methods
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Examination, 11th of May, 2010 from 13 to 16 o'clock.

In order to pass the course and earn 5 ECTS credit points, you must also pass the term project. Results of this examination are valid for one year after the examination date. Information for Finnish speakers: Voit vastata kysymyksiin myös suomeksi, kysymykset on ainoastaan englannin kielellä. Information for Swedish speakers: Du får också svara på svenska, frågorna finns dock endast på engelska.

1. Define the following terms shortly:

a) conditional independence
b) factor graph
c) d-separation
d) Hidden markov model (HMM)
e) Hybrid Monte Carlo algorithm
f) Kalman filter

2. Show how a second-order Markov chain of 3 symbols can be transformed to a Hidden Markov Model with 9 states and 3 symbols. Hint: the joint distribution of the second-order Markov chain is

$$p(x_1, \ldots, x_N) = p(x_1)p(x_2 \mid x_1) \prod_{n=3}^{N} p(x_n \mid x_{n-1}, x_{n-2})$$

3. What is the difference between rejection sampling and importance sampling? Write down the rejection rules and explain the difference.

4. Write the probability $p(\mathbf{x})$ for the finite mixture model of exponential distributions, name the parts of the mixture model, and derive the E-step and the M-step of the Expectation-Maximization (EM) algorithm. Hint: The probability for an exponentially distributed random variable can be calculated with the following equation: $p(x \mid \lambda) = \lambda e^{-\lambda x}, x \geq 0$.

5. For the Bayesian network that decomposes the joint probability as in $p(x_1, \ldots, x_5) = p(x_1)p(x_2|x_1)p(x_3|x_1)p(x_4|x_2, x_3)p(x_5|x_4)$, draw the corresponding graphical representation. Assuming all the variables have discrete values $x_i \in \{0, 1, 2, 3\}$, give the sizes of the tables representing the probabilities for the conditional probability distributions. Moreover, derive the junction tree representation (and name the steps). Draw the resulting junction tree.