Exam: Introduction to Geodesy 07.03.2008

(Function calculator)

1. **Fundamentals**

   (a) Describe the two principal methods of representation of numerical terrain models, and their advantages and disadvantages.

   (b) Give two reasons why gravity is less at the equator than at the poles, and one reason that makes it nevertheless a little stronger.

2. **Statistics, units**

   (a) A plane triangle has three angles, $\alpha = 62^\circ.20 \pm 0^\circ.02$, $\beta = 67^\circ.57 \pm 0^\circ.02$ and $\gamma = 50^\circ.06 \pm 0^\circ.02$. Calculate the sum of the measured angles and its uncertainty (mean error) using propagation of variances. You may assume the angle measurements to be statistically independent, i.e., uncorrelated.

   (b) What do you think, has a gross error occurred in these measurements? Why?

   (c) Given the stochastic variable $\underline{x}$, the probability density distribution of which isCompute the *expected value* of $\underline{x}$. The formula for expected value is

   $$E\left\{\underline{x}\right\} = \int_{-\infty}^{+\infty} x \cdot p\left(x\right) dx.$$

3. **Measurement instruments and methods**

   (a) Explain the self-levelling (automatic) level (drawing!)

   (b) In the Torne river valley the heights of a point in the Finnish and the Swedish precise levelling systems differ by approx. 17 cm. Explain the reasons for the difference.

4. **First and second geodetic problems**

   (a) Given a point A: $x_A = 6\,650\,000\,\mathrm{m}$, $y_A = 480\,000\,\mathrm{m}$. The distance to point B is $s = 2828.472\,\mathrm{m}$ and the azimuth (direction angle) $t = 50\,\mathrm{gon}$. Solve the first (forward) geodetic problem for points A, B.

   (b) Given is also point C with coordinates $x_C = 6\,649\,000\,\mathrm{m}$, $y_C = 479\,000\,\mathrm{m}$. Solve the second (inverse) geodetic problem for the points A, C.

5. **Helmert transformation**

   A special case of Helmert transformation : $\theta = 0$:

   $$\begin{bmatrix} x' \\ y' \end{bmatrix} = (1+m) \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}.$$

   (a) Give the above Helmert transformation's *inverse* transformation. I.e., if

   $$\begin{bmatrix} x \\ y \end{bmatrix} = (1+\widetilde{m}) \begin{bmatrix} x' \\ y' \end{bmatrix} + \begin{bmatrix} \widetilde{\Delta x} \\ \widetilde{\Delta y} \end{bmatrix},$$

   calculate the parameters $\widetilde{m}, \widetilde{\Delta x}, \widetilde{\Delta y}$ expressed into the original parameters $m, \Delta x, \Delta y$ ($\widetilde{\theta}$, like θ, vanishes).

(b) Write the equation into the form

$$\begin{bmatrix} x' - x \\ y' - y \end{bmatrix} = \begin{bmatrix} ? & ? & ? \\ ? & ? & ? \end{bmatrix} \begin{bmatrix} m \\ \Delta x \\ \Delta y \end{bmatrix},$$

I.e., fill in the question marks.

Points:

Question	1 a b	2 a b c	3 a b	4 a b	5 a b	Total
Points	5 2 3	6 2 2 2	5 2 3	4 2 2	5 3 2	25

Points	10	13	16	19	23
Grade	1	2	3	4	5