Exam: Introduction to Geodesy (Maa-6.1213) 28.10.2008

(Function calculator)

1. **Fundamentals**

   (a) What is a clothoid, and why is it being used for building railroads and motorways?

   (b) Describe the scientific controversy that the French Academy's grade measurement expeditions to Lapland and Peru attempted to settle, and how they did it.

2. **Statistics, units**

   (a) A plane triangle has three angles, $\alpha = 72°.12 \pm 0°.01$, $\beta = 67°.32 \pm 0°.01$ and $\gamma = 40°.06 \pm 0°.01$. Calculate the sum of the measured angles and its uncertainty (mean error) using propagation of variances. You may assume the angle measurements to be statistically independent, i.e., uncorrelated.

   (b) What do you think, has a gross error occurred in these measurements? Why?

   (c) Given the stochastic variable $\underline{x}$, the probability density distribution of which is

$$p(x) = \begin{cases} 0 & x < -1 \\ 1 & -1 \leq x \leq 1 \\ 0 & x > 1 \end{cases}$$

   Compute the *expected value* of $\underline{x}$. The formula for expected value is

$$E\{\underline{x}\} = \int_{-\infty}^{+\infty} x \cdot p(x)\, dx.$$

3. **Measurement instruments and methods**

   (a) In the Torne river valley the heights of a point in the Finnish and the Swedish precise levelling systems differ by approx. 17 cm. Explain the reasons for the difference.

   (b) Explain the self-levelling (automatic) level (drawing!)

4. **First and second geodetic problems**

   (a) Given a point A: $x_A = 6\,650\,000\,\mathrm{m}$, $y_A = 500\,000\,\mathrm{m}$. The distance to point B is $s = 2828.472\,\mathrm{m}$ and the azimuth (direction angle) $t = 50\,\mathrm{gon}$. Solve the first (forward) geodetic problem for points A, B.

   (b) Given is also point C with coordinates $x_C = 6\,649\,000\,\mathrm{m}$, $y_C = 499\,000\,\mathrm{m}$. Solve the second (inverse) geodetic problem for the points A, C.

5. **Helmert transformation**

   (a) Given are points' A, B coordinates in the coordinate system (1):

$$x_A^{(1)} = 0\,\mathrm{m},\ y_A^{(1)} = 0\,\mathrm{m}, x_B^{(1)} = 3000\,\mathrm{m}, y_B^{(1)} = 2000\,\mathrm{m};$$

   and in the coordinate system (2):

$$x_A^{(2)} = 3500\,\mathrm{m};\ y_A^{(2)} = 1500\,\mathrm{m};\ x_B^{(2)} = 6500.03\,\mathrm{m};\ y_B^{(2)} = 3500.02\,\mathrm{m}.$$

Assuming that the transformation between systems (1) and (2) is a Helmert transformation:

$$\begin{bmatrix} x^{(2)} \\ y^{(2)} \end{bmatrix} = K \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x^{(1)} \\ y^{(1)} \end{bmatrix} + \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix},$$

calculate its parameters K, θ, Δx and Δy.

(b) What is the inverse matrix of the rotation matrix

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

? Is this matrix ever singular?

Points:

Question	1	2	3	4	5	Total
	a b	a b c	a b	a b	a b	
Points	5	5	5	5	5	25
	2 3	2 1 2	2 3	2 3	2 3	

Points	10	13	16	19	23
Grade	1	2	3	4	5