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EXAM
31.8.2010

S-72.3410 Coding Methods

1. (6p.) Consider the (2,1,3) convolutional code with $\mathbf{G}(D) = \begin{bmatrix} 1+D^2 & 1+D+D^2+D^3 \end{bmatrix}$.
   (a) Find the GCD (greatest common divisor) of its generator polynomials.
   (b) Draw the encoder state diagram.
   (c) Find an infinite-weight information sequence that generates a code word of finite weight.
   (d) Is this code catastrophic or non-catastrophic?

2. (6p.) Consider the 4-ary code $\mathbf{C}$ defined by the following parity-check matrix, where α is primitive in GF(4).

$$\mathbf{H} = \begin{bmatrix} \alpha & \alpha^2 & 1 & 1 \\ \alpha^2 & \alpha & 1 & 0 \end{bmatrix}$$

   Find a systematic generator matrix for the code $\mathbf{C}$. Also write out the 16 codewords in $\mathbf{C}$.

3. Consider a narrow-sense Reed-Solomon code of length 15 and design distance 3.
   (a) (3p.) Compute a generator polynomial for this code.
   (b) (1p.) Determine the rate of this code.
   (c) (2p.) Construct generator and parity-check matrices for this code.

   Hint: $f(x) = x^4 + x + 1$ is a primitive polynomial in $\mathrm{GF}(2)[x]$.

4. A (7,4) Hamming code has the following set of codewords:

0000000	1101000	0110100	1011100
0011010	1110010	0101110	1000110
0001101	1100101	0111001	1010001
0010111	1111111	0100011	1001011

   (a) (2p.) Construct a parity-check matrix for this code.
   (b) (2p.) Draw a Tanner graph corresponding to the parity-check matrix obtained in part (a).
   (c) (2p.) This code is also cyclic. What is the generator polynomial of this code?