S.72-1140 Transmission Methods in Telecommunication Systems

Closed-book Exam on Monday 10.5.2010

1. Amplitude probability density function of Rayleigh distributed fading radio signal envelope is given by $p(r)=\frac{r}{\sigma^2}\exp\left(\frac{-r^2}{2\sigma^2}\right)$, where σ^2 is the variance.

   a) Derive the respective cumulative distribution function.
   b) Find the percentage of time that the signal is 10 dB or more below the RMS value.

2. For a (6,3) systematic linear block code, the three parity-check digits c_4, c_5 and c_6 are:

$$c_4 = d_1 + d_2 + d_3$$
$$c_5 = d_1 + d_2$$
$$c_6 = d_1 + d_3$$

Construct the respective generator matrix for this code.

3. Noise with $\eta = \frac{1}{4}\frac{W}{Hz}$ is applied to a lowpass filter with $H(f)=\frac{1}{1+jf}$. Determine noise power at the output of the filter at bandwidth $B_T = 2\,\text{Hz}$.

4. An angle modulated signal with the carrier frequency $\omega_c = 2\pi\cdot 10^6$ is described by $x(t)=10\cdot\cos[\omega_c t + 0.1\cdot\sin(\pi 2000 t)]$.

   a) Find the average power of the signal when the impedance (resistance) level is 1 ohm
   b) Find the respective frequency deviation Δf
   c) Find the respective phase deviation $\Delta\varphi$
   d) Estimate the required transmission bandwidth *for x(t)*

5. You wish to transmit $C = 20\cdot 10^6$ bits/s by using L = 64 level sinc-pulse signaling in a channel with SNR = 56 dB. (a) What is the theoretical minimum bandwidth (based on information theory) for the transmission? (b) What is the actual required bandwidth based on applied sinc-pulse signaling? (c) How many bits are required for each symbol?

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Collection of Formulas

$$C = W_C \cdot \log_2(1+SNR)\ ,\ \begin{cases} r_{max} = 2B_T = r_b / n = r_b / \log_2(L) \\ \Rightarrow r_b = 2B_T \log_2(L), L = 2^n \end{cases}\ ,\ r = n \cdot f_S$$

$$R(\tau) = < x(\tau)x(t+\tau) >$$

$$P_{avg} = < x^2(t) > = \frac{1}{T}\int_0^T x^2(x)dt = R(0)$$

$$V_{RMS} = \sqrt{P_{avg}}\ \Big|_{R=1\Omega}$$

$$P_{dB} = 10\log(P_1/P_2), P_{dB} = 20\log(V_1/V_2), P_{dBm} = 10\log(P_1/1mW),\ \frac{V_g}{V_i} = \frac{Z_g + Z_L}{Z_L}$$

$$\begin{cases} \text{y(t)=Kx(t - t}_d) \\ \Rightarrow \text{Y(f)=F[y(t)]=}\underbrace{\text{Kexp(-j}\omega \text{t}_d)}_{H(f)} X(f) \\ G_y(f) = |H(f)|^2 G_x(f)\ \text{(= output PDF)} \end{cases}\ ,\ \begin{cases} l = d_{min} - 1,\ t = \lfloor l/2 \rfloor, R_C = k/n \le 1 \\ d_{min}\big|_{max} = n - k + 1\ \text{(repetition codes)} \end{cases}$$

$$\begin{cases} N_R = \int_{-\infty}^{\infty} (\eta/2)|H_R(f)|^2 df \\ \quad = \int_{B_T} (\eta/2)df + \int_{B_T} (\eta/2)df = \eta B_T \end{cases} \begin{cases} P(n,k) = \binom{n}{k} \alpha^k (1-\alpha)^{n-k} \\ \binom{n}{k} = \frac{n!}{k!(n-k)!} \end{cases}$$

$$\begin{cases} B_T \approx 2|D-1|W, 1 >> D >> 1 \\ \beta = A_m f_\Delta / f_m \big|_{A_m = 1, f_m = W} = f_\Delta / W \equiv D \\ B_{T,DSB} = 2W, B_{T,SSB} = W \end{cases}$$

$$\begin{cases} x_C(t) = A_C \cos(\omega_C t + \phi(t)) \\ \phi_{PM}(t) = \phi_\Delta x(t) \\ \phi_{FM}(t) = 2\pi f_\Delta \int_{t_0}^{t} x(\lambda)d\lambda, t \ge t_0 \end{cases} \quad \phi(t) = \begin{cases} \underbrace{\phi_\Delta A_m}_{\beta} \sin(\omega_m t), \text{PM} \\ \underbrace{(A_m f_\Delta / f_m)}_{\beta} \sin(\omega_m t), \text{FM} \end{cases} \begin{cases} \gamma = S_R / (\eta W) \\ S_R / N_R = \gamma W / B_T \\ \gamma_b = E_b / N_0 \end{cases}$$

$$y(t) = \begin{cases} v_i(t) & \text{Synchronous detector} \\ A_v(t) - \overline{A_v} & \text{Envelope detector} \\ \phi_v(t) & \text{Phase detector} \\ d\phi_v(t)/dt & \text{Frequency detector} \end{cases}\ ,\ \begin{cases} x_{AM}(t) = A_C[1 + \mu x_m(t)]\cos(\omega_c t) \\ x_{DSB}(t) = x_m(t)\cos(\omega_c t) \end{cases}\ \ \mu = \frac{A_{max} - A_{min}}{A_{max} + A_{min}}$$

$$\begin{cases} Q = R\sqrt{C/L} \\ f_0 = (2\pi\sqrt{LC})^{-1} \end{cases}, \quad H(\omega) = V_{out}(\omega) / V_{in}(\omega) = z_p / z_i$$

$$Q(k) = \frac{1}{\sqrt{2\pi}} \int_k^{\infty} \exp\left(-\frac{\lambda^2}{2}\right) d\lambda$$

$$\sigma^2 + \mu^2 = \int_A p(x,\mu,\sigma) x^2 dx, \mu = \int_A p(x,\mu,\sigma) x dx$$

$$\lambda = (m - x) / \sigma \Rightarrow Q(k) = \frac{1}{\sqrt{2\pi}} \int_{\sigma k+m}^{\infty} \exp\left(-\frac{(x-m)^2}{2\sigma^2}\right) dx$$

$$\begin{cases} P = UI = U^2 / R = I^2 R \\ R = U / I \end{cases}, \quad \frac{V_g}{V_i} = \frac{Z_g + Z_L}{Z_L}, P_L = V_i I_i \cos\theta$$

$$\cos\theta = R_{tot} / Z_{tot} = R_{tot} / \sqrt{R_{tot}{}^2 + X_{tot}^2}, X_{tot} = X_g + X_L, R_{tot} = R_L + R_g$$

$$N_{D(PM)} = \int_{-W}^{W} \frac{\eta}{2S_R} df = \frac{\eta W}{S_R}, \quad N_{D(FM)} = \int_{-W}^{W} \frac{\eta f^2}{2S_R} df = \frac{\eta W^3}{3S_R}$$

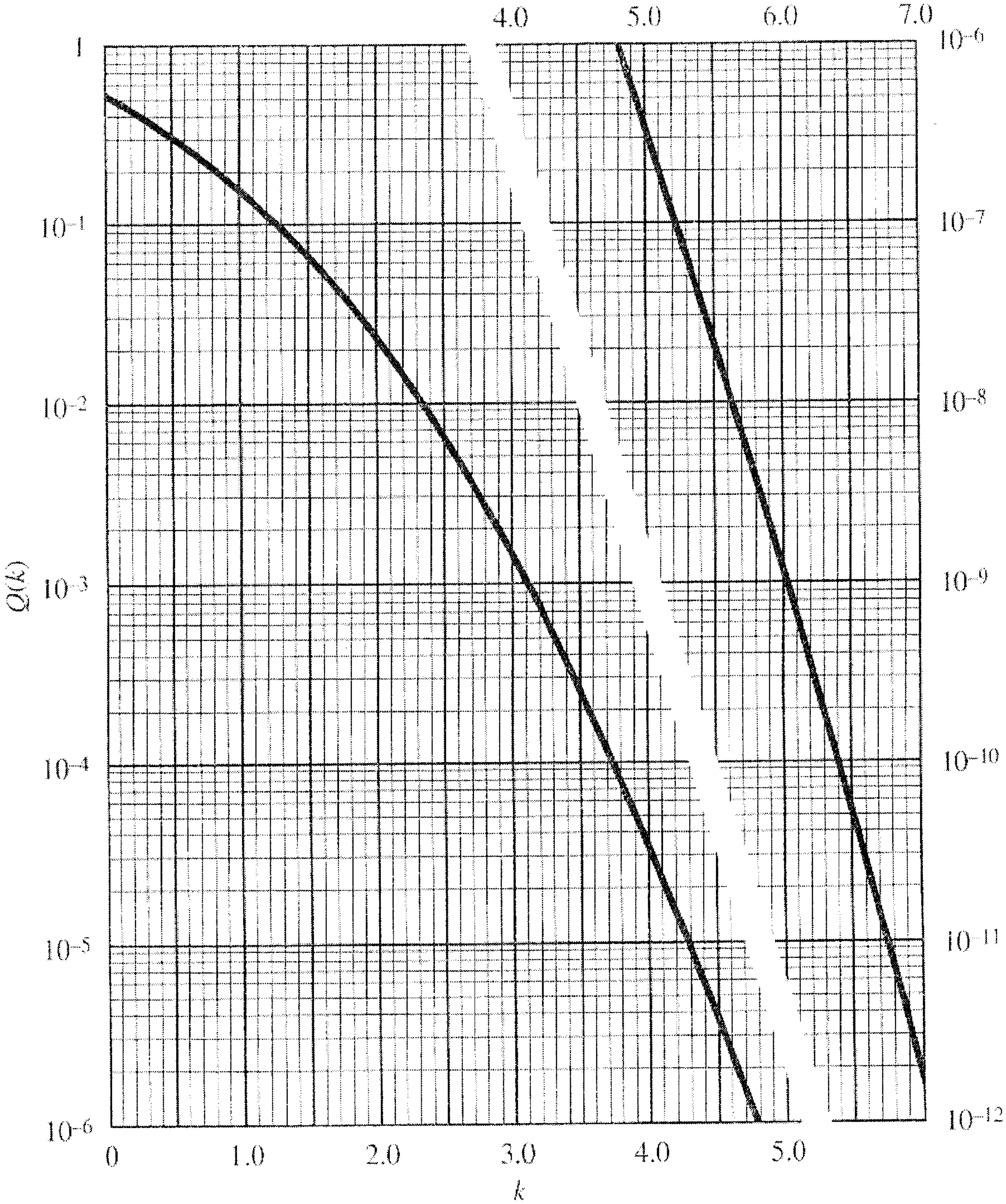
$$S_D / N_D\big|_{FM} = \frac{f_\Delta^2 S_x}{\eta W^3 / (3S_R)} = 3\underbrace{\left(\frac{f_\Delta}{W}\right)}_{D}^2 S_x \underbrace{\frac{S_R}{\eta W}}_{\gamma} = 3D^2 S_x \gamma, \quad S_D / N_D\big|_{FM,D>>1} = \frac{3}{4}\left(\frac{B_T}{W}\right)^2 S_x \gamma$$

$$S_D / N_D\big|_{PM} = \frac{\phi_\Delta^2 S_x}{\eta W / S_R} = \phi_\Delta^2 S_x \gamma, \text{where } \phi_\Delta^2 S_x \le \pi^2$$

$$\begin{cases} \int \frac{1}{1+x^2} dx = \arctan(x) \\ \int \frac{x^2}{1+x^2} dx = x - \arctan(x) \end{cases} \quad \begin{cases} \prod\left(\frac{t}{\tau}\right) \leftrightarrow \tau \operatorname{sinc} f\tau \\ \Lambda\left(\frac{t}{\tau}\right) \leftrightarrow \tau \operatorname{sinc}^2 f\tau \end{cases}$$

$$\begin{cases} \frac{d^n v(t)}{dt^n} \leftrightarrow (j2\pi f)^n V(f) \\ \int_{-\infty}^{t} v(\lambda) d\lambda \leftrightarrow \frac{1}{j2\pi f} V(f) + \frac{1}{2} V(0)\delta(f) \end{cases}, \quad \begin{aligned} &\int g'(f(x)) f'(x) dx = g(f(x)) + C \\ &\int f(x) g'(x) dx = f(x) g(x) - \int g(x) f'(x) dx \end{aligned}$$

$$\begin{cases} \sin\alpha \sin\beta = 1/2\cos(\alpha-\beta) - 1/2\cos(\alpha+\beta) \\ \cos\alpha \cos\beta = 1/2\cos(\alpha-\beta) + 1/2\cos(\alpha+\beta) \\ \sin\alpha \cos\beta = 1/2\sin(\alpha-\beta) + 1/2\sin(\alpha+\beta) \end{cases}, \quad \begin{cases} \cos^2\alpha = (1+\cos 2\alpha)/2 \\ \cos^3\alpha = (3\cos\alpha + \cos 3\alpha)/4 \end{cases} \quad \begin{cases} (\alpha+\beta)^2 = \alpha^2 + 2\alpha\beta + \beta^2 \\ (\alpha+\beta)^3 = \alpha^3 + 3\alpha^2\beta + 3\alpha\beta^2 + \beta^3 \end{cases}$$



Q(k)
1
10⁻¹
10⁻²
10⁻³
10⁻⁴
10⁻⁵
10⁻⁶
0
1.0
2.0
3.0
4.0
5.0
k
4.0
5.0
6.0
7.0
10⁻⁶
10⁻⁷
10⁻⁸
10⁻⁹
10⁻¹⁰
10⁻¹¹
10⁻¹²
Q(k)

TABLE A6.5 *Table of Bessel functions*[a]

				$J_n(x)$					
n\x	*0.5*	*1*	*2*	*3*	*4*	*6*	*8*	*10*	*12*
0	0.9385	0.7652	0.2239	−0.2601	−0.3971	0.1506	0.1717	−0.2459	0.0477
1	0.2423	0.4401	0.5767	0.3391	−0.0660	−0.2767	0.2346	0.0435	−0.2234
2	0.0306	0.1149	0.3528	0.4861	0.3641	−0.2429	−0.1130	0.2546	0.0849
3	0.0026	0.0196	0.1289	0.3091	0.4302	0.1148	−0.2911	0.0584	0.1951
4	0.0002	0.0025	0.0340	0.1320	0.2811	0.3576	−0.1054	−0.2196	0.1825
5	—	0.0002	0.0070	0.0430	0.1321	0.3621	0.1858	−0.2341	−0.0735
6		—	0.0012	0.0114	0.0491	0.2458	0.3376	−0.0145	−0.2437
7			0.0002	0.0025	0.0152	0.1296	0.3206	0.2167	−0.1703
8			—	0.0005	0.0040	0.0565	0.2235	0.3179	0.0451
9				0.0001	0.0009	0.0212	0.1263	0.2919	0.2304
10				—	0.0002	0.0070	0.0608	0.2075	0.3005
11					—	0.0020	0.0256	0.1231	0.2704
12						0.0005	0.0096	0.0634	0.1953
13						0.0001	0.0033	0.0290	0.1201
14						—	0.0010	0.0120	0.0650