

S-72.2410 Information Theory

1. (6p.) There are two biased coins in a box. The first coin when flipped will produce a “head” with probability $3/4$, while the second coin will produce a “head” with probability $1/4$. A coin is randomly selected from the box and flipped. What is the average mutual information provided about the coin selected when the outcome of a flip of a randomly selected coin is observed?
2. (6p.) A discrete memoryless source has an alphabet of eight letters x_i , $i = 1, 2, \dots, 8$, with probabilities 0.25, 0.20, 0.15, 0.12, 0.10, 0.08, 0.05, and 0.05. Use the Huffman encoding procedure to determine a quaternary code (using symbols 0, 1, 2, and 3) for encoding the source output. What is the entropy of the source? What is the expected code length?
3. A dog walks on the integers, possibly reversing direction at each step with probability $p = 0.1$. Let $X_0 = 0$. The first step is equally likely to be positive or negative. A typical walk might look like this:

$$(X_0, X_1, \dots) = (0, 1, 2, 3, 4, 3, 2, 1, 0, 1, \dots).$$

- (a) (2p.) Find $H(X_1, X_2, \dots, X_n)$.
 - (b) (2p.) Find the entropy rate of this browsing dog.
 - (c) (2p.) What is the expected number of steps the dog takes before reversing direction?
4. Consider a commander of an army besieged a fort for whom the only means of communication to his allies is a set of carrier pigeons. Assume that each carrier pigeon can carry one ASCII character (7 bits), and assume that pigeons are released once every 5 minutes, and that each pigeon takes exactly 3 minutes to reach its destination.
 - (a) (2p.) Assuming all the pigeons reach safely, what is the capacity of this link in bits/hour?
 - (b) (2p.) Now assume that the enemies try to shoot down the pigeons, and that they manage to hit a fraction α of them. Since the pigeons are sent at a constant rate, the receiver knows when the pigeons are missing. What is the capacity of this link?
 - (c) (2p.) Now assume that the enemy is more cunning, and every time they shoot down a pigeon, they send out a dummy pigeon carrying a random character (chosen uniformly from all 7-bit characters). What is the capacity of this link in bits/hour?

Set up an appropriate model for the channel in each of the above cases, and indicate how to go about finding the capacity.