HUT, Department of Mathematics and Systems Analysis
Mat-1.2620 Applied Probability B
Fall 2009, Talponen/Ruokokoski
First MIDTERM EXAM Answer the following problems.

Problem 1. Consider two urns, both of which contain 3 black and 5 white balls.

(a) Suppose you pick randomly one ball from each urn. What is the probability that both of them are black?

(b) Suppose you pick randomly two balls from one of the urns. Again, what is the probability that both of them are black?

Problem 2. In urn A there are 4 white and 6 black balls and in urn B there are 6 white and 4 black balls. You randomly pick a ball from each urn and put the ball picked from urn A into urn B and the ball picked from urn B into urn A. After this you randomly pick one more ball from urn B. What is the probability that this ball is white? Use a probability tree.

Problem 3. Let $X_1, X_2, \ldots, X_n$, $n = 10^4$, be independent, identically distributed random variables with expectation 0 and variance 1 (and we don't have any other information about the distributions). Define a new random variable Y by taking the average of the abovementioned random variables. Give a good approximation of the probability

$$\Pr(0 \leq Y \leq 1).$$

Hint: You might want to begin solving the problem by calculating the expectation and variance of Y.

Problem 4. Answer questions i) and ii).
i) Let X and Y be two-dimensional non-correlated random variables. Are they necessarily independent? (Justify your answer by a calculation or a counter example.)
ii) Let $\Pr(A) = 0.6$ and $\Pr(B) = 0.2$. Calculate the probability of $A \cup B$ or detect an impossible situation in the following cases:

(a) A and B are mutually exclusive,

(b) A and B are independent,

(c) $\Pr(A \cap B) = 0.3$,

(d) $\Pr(A|B) = 0.6$,

(e) $\Pr(B|A) = 0.6$,

(f) $\Pr(A|B) = \Pr(B|A)$.