T-61.3040 Statistical Modeling of Signals

Final Exam 3.9.2009

In the exam you are allowed to have a calculator (non-programmable or memory emptied) and basic mathematical tables (no tables containing material directly associated with the course). For example, the book "BETA Mathematics Handbook for Science and Engineering" by Råde & Westergren contains material that is too directly associated with the course; therefore you are NOT allowed to have that book in the exam. The results of the exam will be announced eventually through the Noppa system.

1. (max 6p)
Explain *briefly* the following topics without unnecessary detail:
i) LMS algorithm (2p)
ii) Wide-sense stationarity (WSS) (2p)
iii) Wold decomposition (2p)

2. (max 6p)
You have observed the following values of the real-valued WSS process $x(n)$: $x(0) = 4$, $x(1) = -2$, $x(2) = 2$.
i) Estimate an autocorrelation matrix of size 3×3 so that the result is positive semidefinite. Show that the result is positive semidefinite. (2p)
ii) Model the process $x(n)$ as an AR(2) process. (2p)
iii) What are the variance $\mathrm{Var}(x(n))$ and conditional variance $\mathrm{Var}(x(3)|x(2), x(1), x(0))$ of the modeled process? (2p)

3. (max 6p)
Answer the following propositions either "true" or "false": you may also leave any of them unanswered. A correct answer gives 1 point, a wrong answer -1 points, and a missing answer zero points. However, the total number of points you receive from this problem cannot become negative; the total number of points is at least zero. No need to justify your answers.

a) A strict sense stationary process is always also wide sense stationary.
b) When applying Wiener filtering to the process $x(n) = d(n) + v(n)$, if the noise $v(n)$ is zero-mean white noise that is uncorrelated with the desired signal $d(n)$, then the Wiener filter can be solved using only the autocorrelation $r_v(k)$ of the noise and the autocorrelation $r_d(k)$ of the desired signal.
c) The resolution of the periodogram improves as the number of data grows.
d) Using a Wiener filter we are able to remove the noise entirely from the signal also on the frequency band of the desired signal.
e) If we form two ARMA processes with different parameters, we always get different processes.
f) For both the power spectrum and the pseudospectrum, the integral over some frequency band tells the power of the process on that band.

4. (max 6p)
The process $x(n) = A_1 \exp(jn\omega_1) + A_2 \exp(jn\omega_2) + v(n)$ consists of two complex sinusoids in white noise. The multipliers are of the form $A_1 = |A_1|e^{j\phi_1}$ and $A_2 = |A_2|e^{j\phi_2}$. The phases ϕ_1 and ϕ_2 are random numbers uniformly distributed in the interval $[-\pi, \pi]$ (they are sampled anew for each realization of the process, but each realization uses the same multipliers A_1 and A_2 at different moments n). The autocorrelation matrix of the process is

$$R_x = \begin{bmatrix} 6 & -\sqrt{3}j & -\sqrt{3}j \\ \sqrt{3}j & 6 & -\sqrt{3}j \\ \sqrt{3}j & \sqrt{3}j & 6 \end{bmatrix}$$

It is known that the frequencies ω_1 ja ω_2 are unequal ($\omega_1 < \omega_2$), and they have been chosen from three alternatives: $\pi/6$, $\pi/3$, and π. It is also known that the smallest eigenvalue of the matrix is 3 and the corresponding eigenvector is

$$C \cdot \begin{bmatrix} 1+\sqrt{3}j \\ 2 \\ a \end{bmatrix}$$

where a is some complex number and the multiplier C normalizes the length of the vector to be 1.

i) Using Pisarenko's method, solve which of the three possible frequencies are the true frequencies ω_1 and ω_2 of the sinusoids. (4.5p)
ii) Now you know the frequencies of the sinusoids. You observe one realization of the process, and get from it the observations $x(0)$ and $x(1)$. Explain how you can estimate the multipliers A_1 and A_2 based on the observations. (1.5p)

Equations you might need:

ω	$\sin(\omega)$	$\cos(\omega)$
$\pi/6$	$1/2$	$\sqrt{3}/2$
$\pi/3$	$\sqrt{3}/2$	$1/2$
$2\pi/3$	$\sqrt{3}/2$	$-1/2$