HELSINKI UNIVERSITY OF TECHNOLOGY
Networking Laboratory
S-38.3143 Queueing Theory

Examination
11.1.2008
Virtamo

Mark clearly (A/B) wheteher you would like to have the course registered A) with the new code 38.3143 (5 ETCS pt) or B) old code 38.143 (3 cr).

1. Let $N(t)$, $t \geq 0$, be a Poisson process with rate λ. Let S_n denote the time of occurance of the nth event. Find

   a) $\mathrm{E}[S_4]$

   b) $\mathrm{E}[S_4 | N(2) = 2]$

   c) $\mathrm{E}[N(4) - N(2) | N(1) = 4]$

2. When a customer A arrives to a bank all four service counters are reserved and furthermore there are two customers in front of him in queue (one common queue to all counters). The service time of each customer is assumed to obey exponential distribution with mean 1 min.

   a) What is the probability that when customer A finally leaves the bank all those customers who arrived before him have already left?

   b) How long does it take on average before customer A gets to a counter?

   c) What is the mean sojourn time of customer A in bank?

3. Empty taxis pass a street corner at a Poisson rate of 2 per minute and pick a passenger if one is waiting there. Passengers arrive at the street corner at rate 1 per minute and wait for a taxi only if there are fewer than four persons waiting; otherwise they leave and never return. Find the average waiting time of a passenger who joins the queue.

4. a) Give the Pollaczek-Khinchin mean formula for the waiting time in an $M/G/1$ system.

   b) Customers arrive at an $M/G/1$ queue according to a Poisson process with intensity λ. The service of each customer consists of k consecutive tasks (only first all the tasks have been accomplished the next customer can enter the server). Each of the tasks has duration obeying the distribution $\mathrm{Exp}(\mu)$ independent of the others. Calculate the average waiting time of a customer.

5. In the closed queueing network depicted in the figure there are three customers. What is the average cycle time of a customer and the average customer stream through queue 4?