Mat-1.2600 Applied Probability A

1-st Intermediate Examination, 30 October 2009 / Mellin

Write ***clearly on every sheet of paper*** the following information:

- Mat-1.2600 ApplProbA 1-st Intermediate Examination, 30 Oct 2009
- Your student identification number
- IN BLOCK LETTERS IN THE FOLLOWING ORDER:
  your family name, your first name(s)
- Your degree programme, the year of registration
- Your former names and degree programmes
- Signature

A function calculator **plus** ***the collections of formulae and statistical tables*** **of Mellin are allowed.**

Justify your solutions**: a correct numerical value without any justification** ***will not give points.***

1. (a) A and B play a game consisting of distinct rounds. The probability that A wins a round is 0.6 and the probability that B wins a round is 0.4. The first to win 5 rounds gets 100 €. However, the game will be interrupted, when A has won 3 rounds and B has won 2 rounds.

Question 1: Compute the probability that B will get the prize of 100 €, if the game will be continued the next day.

Question 2: Suggest a fair way to divide the prize, if it will not be possible to continue the game.

(b) A fish monger has 12 oysters, four of the oysters having a pearl. You will randomly pick up 8 oysters.

Question 1: Compute the probability that you will find at least 2 pearls.

Question 2: Compute the expected value of the pearls turning up.

2. The length of the steel rods is a normally distributed random variable with the parameters $\mu = 1$ m and $\sigma^2 = 0.0001$ m^2. Suppose that you will randomly pick up 20 rods with replacement.

(a) Compute the probability that there is at most one rod in the sample the length of which is at most 0.99 m.

(b) Compute the expected number of rods in the sample the length of which being at most 0.99 m.

3. Consider a circuit consisting of 100 similar components connected serially. The life-times of the components are random variables having the exponential distribution Exp($1/\mu$), where $\mu = 0.1$ years.

(a) Question 1: What is the distribution of the lifetime of the circuit?

Question 2: What is the expected life-time of the circuit?

(d) Assume that failed components are immediately replaced with new ones.

Compute the probability that the number of failed components in a year will be less than 981.

4. (a) Let the joint distribution of the random variables X and Y be the bivariate (i.e. two-dimensional) normal distribution. Further, let the regression function of Y on X be

$$4y = -8 - 9x$$

and let the regression function of X on Y be

$$25x = -18 - 9y$$

Compute the correlation of X and Y.

(b) Roll a fair dice twice (a dice is considered to be fair, if each of the spot numbers 1, 2, 3, 4, 5, 6 has equal probability to turn up).

Let

X = the result of the 1-st roll

Y = the result of the 2-nd roll

$U = XY$

Compute the conditional expectation

$$E(X \mid U = 12)$$