S72-3280 Advanced Radio Transmission Methods
Exam 7.01.2008

1. The channel impulse response is $h_{ch} = \begin{bmatrix} \frac{2}{3} & \frac{2}{3} & \frac{1}{3} \end{bmatrix}$. The transmitted symbols are modulated to be either 1 or -1. The energy per bit to noise ratio is $E_b / N_0 = 5\ dB$

   The received symbol sequence is $y_{rec} = \begin{bmatrix} 0.97 & 0.2 & 0.63 & -0.37 & -0.27 \end{bmatrix}$.

   a) Estimate the most probable bit sequence by using Viterbi algorithm.
   b) Explain why the Viterbi decoder actually does not need the information about E_b / N_0.

2. Generator polynomials of the convolutional code (given in binary form) are

   $\mathbf{g}_1 = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$, $\mathbf{g}_2 = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix}$

   a) Sketch the state diagram of the code.
   b) Find the transfer function $T(D)$.
   c) Calculate error probability by approximating the union bound by using only two smallest terms of the code spectrum (code distance)?

3. Combining extrinsic information and the bit information
   The system uses 4 AM modulation with symbols C mapped as seen in figure 1. The channel is modeled as AWGN with SNR= 5 dB. The received signal value is $y = 0.3$.

a) Calculate the aposterior probability for each possible symbol in the constellation. (Assume that there is no information about prior probabilities)
b) We know that the prior probability of the first bit b_1 being 1 is $p_{prior}(b_1 = 1) = 0.8$. About the second bit there is no prior information. What is the aposteriori probability for the second bit being 1 $p_{apos}(b_2 = 1)$.

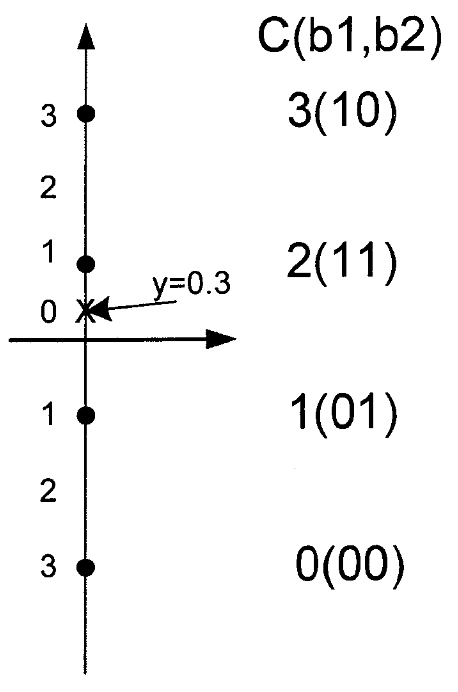

Figure 1. AM constellation.

4. Consider the signal model $\mathbf{y} = \mathbf{Hx}+\mathbf{n}$, where the channel $\mathbf{H}$ is a 2 x 2 matrix. The transmitted symbols are $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, with average power 1, the received signals are $\mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$, and the additive noise is $\mathbf{n} = \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$ with noise power spectral density $N_0 = 1$.
   a. Consider the channel $\mathbf{H} = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$, and a matched filter receiver. What is the post-processing signal-to-interference-plus-noise ratio (SINR) (i.e. SINR after matched filter processing) experienced by x_1 and x_2?
   b. Consider the case where x_1 and x_2 interfere slightly, so that the channel is $\mathbf{H} = \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$. What is the post-processing SINR experienced by x_1 and x_2 assuming a matched filter receiver?
   c. Consider the same channel as in b), but a zero-forcing receiver. What is the post-processing SINR experienced by x_1 and x_2?
   d. Compare the results of a) b) and c). What can you conclude about the effect of interference on the two kinds of receiver?
   e. What would change in your conclusion if $N_0 = 10$?

5. Assume a tapped delay line channel with two channel taps $\begin{bmatrix} h_0 & h_1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \end{bmatrix}$. During time instance t, symbol x_t is transmitted.
   a. Write a matrix signal model describing the received signals $\begin{bmatrix} y_1 & y_2 & y_3 \end{bmatrix}^T$ in time instances [1, 2, 3] in terms of transmitted symbols $\begin{bmatrix} x_0 & x_1 & x_2 & x_3 \end{bmatrix}^T$.
   b. Construct the 3-tap transversal MMSE filter for estimating symbol x_1 from the signal model constructed above. Assume that the noise power spectral density is $N_0 = 1$. You may need to know the inverse matrix

$$\begin{bmatrix} a & b & 0 & 0 \\ b & c & b & 0 \\ 0 & b & c & b \\ 0 & 0 & b & a \end{bmatrix}^{-1} \sim \begin{bmatrix} ac^2 - (a+c)b^2 & b^3 - abc & ab^2 & -b^3 \\ b^3 - abc & a^2c - ab^2 & -a^2b & ab^2 \\ ab^2 & -a^2b & a^2c - ab^2 & b^3 - abc \\ -b^3 & ab^2 & b^3 - abc & ac^2 - (a+c)b^2 \end{bmatrix}$$

   The proportionality constant (the determinant) may be omitted.
   c. Write an equation describing the result of filtering the received signals $\begin{bmatrix} y_1 & y_2 & y_3 \end{bmatrix}^T$. Normalize the filter output so that the coefficient of x_1 is one.
   d. The simplest receiver in this case is a one-tap equalizer, which estimates symbol x_1 directly from y_1. Why does the 3-tap equalizer provide better performance than the one-tap equalizer?