HELSINKI UNIVERSITY OF TECHNOLOGY **Exam 15 Dec. 2004**
Signal Processing Laboratory Timo I. Laakso
S-88.211 Signal Processing in Telecommunications I (2 cu)

Write in each answer paper your name, department, student number, the course name and the date. Number each paper you submit and denote the total no. of pages. The exam paper is only available in English, but please feel free to write in Finnish or Swedish if you prefer. 5 problems, 30 points total. The BETA mathematical tables can be utilized – you can borrow a copy from the exam supervisor if you do not have your own.

1. (1p each) Define and describe *briefly* (2..3 lines of text) the following concepts:
   a) Zero-forcing equaliser
   b) OFDM
   c) Water-pouring theorem
   d) DFE
   e) Nyquist criterion
   f) MLSD

(6p) Define and explain the matched filter for an AWGN channel . Define it in the frequency domain and derive the time-domain form. Explain how it can be combined with Nyquist's ideas for practical Tx and Rx filter design with excess bandwidth.

2. Capacity of mobile communications

In mobile communications, the channel may vary with time, distance etc. Let us assume that the channel response is a real-valued constant, i.e. $C(f) = K$ and the channel noise is AWGN with the power spectrum $S_n(f) = P_n/(2W)$. $W = 4$ kHz and $SNR = P_x/P_n = 5$ dB.

a) (3p) Solve for the optimum transmit power spectrum $S_x(f)$ that maximizes the channel capacity when the total transmit power P_x is limited.

b) (2p) Determine the general expression for the channel capacity and the numerical value in this case (assume $K = 1$).

c) (1p) The channel fluctuations are modeled by assuming that K takes values 0.1, 1, and 10 equally often. Determine the average channel capacity.

Hints: The optimal power spectrum is obtained with the water-pouring theorem as

$$S_{x,opt}(f) = L - S_n(f)/|C(f)|^2 \qquad (1)$$

where L is determined so that the total transmit power

$$P_x = \int_{-\infty}^{\infty} S_{x,opt}(f)df \qquad (2)$$

is limited. The capacity is then obtained by integration

$$C = \int_0^\infty \log_2\left(1 + \frac{S_x(f)|C(f)|^2}{S_n(f)}\right)df = \frac{1}{2}\int_{-\infty}^{\infty} \log_2\left(1 + \frac{S_x(f)|C(f)|^2}{S_n(f)}\right)df \tag{3}$$

4. Adaptive filters. Let us consider a discrete-time model for a communication system in a linear channel (sampled at symbol rate). The received signal samples $r(k)$ are filtered by an N-tap FIR filter (equalizer). The equalizer output can be expressed as

$$y(k) = \mathbf{h}_\mathrm{R}^\mathrm{T}\mathbf{r}(k). \tag{4}$$

where $\mathbf{h}_\mathrm{R}$ and $\mathbf{r}$ are N-dimensional column vectors. The mean squared error (MSE) can be expressed as

$$E\left[e^2(k)\right] = E\left[a_k^2\right] - 2\mathbf{h}_\mathrm{R}\mathbf{p}_0 + \mathbf{h}_\mathrm{R}^\mathrm{T}\mathbf{R}\mathbf{h}_\mathrm{R} \tag{5}$$

where $\mathbf{p} = E\left[\mathbf{r}(k)a_k\right]$ and $\mathbf{R} = E\left[\mathbf{r}(k)\mathbf{r}^\mathrm{T}(k)\right]$.

a) (2p) Derive the optimal minimum-MSE equalizer.
b) (2p) The general structure for an adaptive equalizer using *MSE gradient* (MSEG) algorithm is of the form

$$\mathbf{h}_\mathrm{R}[j+1] = \mathbf{h}_\mathrm{R}[j] - \frac{\beta}{2}\nabla_{h_\mathrm{R}} \mathrm{E}\left[e^2(k)\right]. \tag{6}$$

Derive the MSEG algorithm for the equalizer.

c) (2p) In practice, usually we do not know $\mathbf{p}$ or $\mathbf{R}$. However, we can assume that we have access to a reference signal containing the correct symbol sequence a_k (e.g., a known training signal or decided symbols) so that we can compute the error

$$e(k) = y(k) - x(k). \tag{7}$$

Derive the *stochastic gradient* (SG) algorithm for the filter which uses the *instantaneous* squared error to determine the gradient estimate.

5. EXTRA (9p total)

Let us derive pulse waveforms which meet the Nyquist criterion:

a) (3p) Assume an ideal baseband communication system of data rate $1/T$ in an AWGN channel which needs no excess bandwidth. Define the *pulse spectrum* and derive the *continuous-time pulse waveform* via inverse Fourier transform. No filtering assumed in the receiver, just symbol-rate sampling.
b) (3p) The same as a) except now we assume excess bandwidth α ($0< \alpha <1$) and that the spectrum is *piecewise constant.*
c) 3p) The same as b) except now we want to design transmit and receive filters that form a matched-filter pair and whose *convolution meets the Nyquist criterion.* Assume that the *spectrum of the convolution is piecewise constant.*