S-72.245 TRANSMISSION METHODS IN TELECOMMUNICATION SYSTEMS (4 CR)

Exam 21.12.2004

REMARKS/HUOMAUTUKSET:

- Please see the formula collection overleaf of this paper/Huomaa, että tämän arkin kääntöpuolella on kaavakokoelma
- Return all answer papers – even empty ones/Palauta kaikki saamasi vastauskonseptit – myös tyhjät sellaiset
- All kinds of calculators are allowed/Voit käyttää millaista laskinta tahansa
- You can answer in English, Swedish or Finnish/Du kan svara på svenska/Voit vastata suomeksi
- Please show your identity when leaving papers to supervisor/Ole hyvä ja osoita henkilöllisyytesi jättäessäsi paperit valvojalle

This is closed-book exam – no handouts or other references can be present

1. Signal $f(t)=\exp(-at)u(t)$, a>0, where $u(t)$ is the unit step function, is not strictly band limited, and therefore its sampling results some aliasing or spectral folding. Due to its low-pass characteristics, aliasing will occur at the frequencies $\omega_s/2\le\omega<\omega_s$, where ω_s is the sampling frequency. Determine the minimum value of ω_s in terms of the constant *a* in $f(t)$:s expression so that the signal energy in the range $\omega_s/2\le\omega<\omega_s$ is down at least 10 dB from the energy in the range of $0\le\omega<\omega_s/2$.

2. Tone modulation is applied simultaneously to a frequency modulator and phase modulator and the two output spectra are identical. Describe how these two spectra will change when: (a) The tone amplitude is increased or decreased (b) The tone frequency is increased or decreased.

3. Determine the (a) average power, (b) autocorrelation for $f(t)=\sin(\alpha t)+\sin(\alpha t+\phi)$. (c) <u>Generally</u>, what is the relationship of (a) and (b) and why?

4. (a) Show that linear channel can not generate new frequency components that don't exist in its input. (b) Explain the difference of soft decision and hard decision! (both 3 lines of text/formulas max.)

5. (a) Sketch a block diagram for a DSB modulator by using a non-linear element with $v_{out}=a_1v_{in}+a_3v_{in}^3$. (b) What is the condition of the carrier frequency f_c in terms of the modulating signal v_{in} and the modulating signal bandwidth *W*? *Hint:* consider $v_{in}(t)=x(t)+\sin\omega_0 t$.

Collection of formulas:

$$d\phi(t)/dt = 2\pi f(t) = 2\pi[f_C + f_\Delta x(t)]\ ,\ d\phi(t)/dt = [\phi(t) - \phi(t - t_1)]/t1$$

$$\phi(t) - \phi(t - t_1) = t_1 d\phi(t)/dt = 2\pi t_1[f_C + f_\Delta x(t)]$$

$$x_\delta(t) = x(t)s_\delta(t) = x(t)\sum_{k=-\infty}^{\infty}\delta(t - kT_s),\ = \sum_{k=-\infty}^{\infty} x(kT_s)\delta(t - kT_s)\ ,$$

$$X_\delta(f) = f_s \sum_{n=-\infty}^{\infty} X(f - nf_s),\ F[\exp(-at)u(t)] = (a + j\omega)^{-1}, a > 0$$

$$q = M^v,\ |\varepsilon_k| \le 1/q\ ,\ P_{dB} = 10\log_{10}(P/P_{ref})$$

$$< v_i(t) >= \frac{1}{T}\int_{-T/2}^{T/2} v_i(t)dt, < v_i(t) >= \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} v_i(t)dt$$

$$< v(t) >= \int_A x p_v(x)dx, < v_i^2(t) >= \int_A x^2 p_v(x)dx$$

$$< v_i^2(t) >= \frac{1}{T}\int_{-T/2}^{T/2} v_i^2(t)dt$$

$$\phi(t) = \begin{cases} \underbrace{\phi_\Delta A_m}_{\beta} \sin(\omega_m t), & \text{PM} \\ \underbrace{(A_m f_\Delta / f_m)}_{\beta}\sin(\omega_m t), & \text{FM} \end{cases}$$

$$\sin^2(x) = (1 - \cos(2x)/2\ ,\ \sin^3(x) = \tfrac{1}{4}[3\sin(x) - \sin(3x)]\ ,$$

$$\sin(x)\sin(y) = [\cos(x - y) - \cos(x + y)]/2$$

$$\sin(x)\cos(y) = [\sin(x - y) + \sin(x + y)]/2$$

$$y_D(t) \approx \begin{cases} K_{D,AM} A_v(t), \text{AM} \\ K_{D,PM}\phi(t), \text{PM} \\ K_{DFM}\, d\phi(t)/dt, \text{FM} \end{cases},\ \beta = A_m f_\Delta / f_m\big|_{A_m=1, f_m=W} = f_\Delta / W \equiv D$$

$$\cos(\beta\sin(\omega_m t)) = J_O(\beta) + \sum_{n\ \text{even}}^{\infty} 2J_n(\beta)\cos(n\omega_m t)$$

$$\sin(\beta\sin(\omega_m t)) = \sum_{n\ \text{odd}}^{\infty} 2J_n(\beta)\sin(n\omega_m t)$$

$$x_{PM}(t) = A_c \cos(\underbrace{\omega_C t + \underbrace{\phi(t)}_{\phi_\Delta x(t), \phi_\Delta \le \pi}}_{\theta_C(t)}),\ X_C(f) \approx \frac{1}{2}A_C\delta(f - f_C) + \frac{j}{2}A_C\Phi(f - f_C), f > 0$$

$$C_{xy} = \int_A x(t)y(t - \tau)dt\ , x(t)\otimes y(t) = \int_A x(t)y(\tau - t)dt$$