S-72.2505 Digital Transmission Methods

Exam 15. 12. 2010

All five tasks are evaluated and taken into account in the grading. The exam can be written in Finnish, Swedish or English.

This is a closed book exam

1. The mobile radio channel can be roughly characterized by two numbers, the coherence time, and the coherence bandwidth.

   a) What is the relation of the coherence bandwidth to the delay spread of the channel?

   b) Using the coherence bandwidth and the system bandwidth, how do you define a frequency flat vs. frequency selective fading channel?

   c) What is the essential difference when designing a communication system for a frequency selective as opposed to a frequency flat channel?

   d) What is the relation of the coherence time to the Doppler spread of the channel, and what is the relation of Doppler spread to speed?

   e) Using the coherence time and the symbol period, how do you define a slow vs. rapidly fading channel?

   f) What is the essential difference when designing a communication system to operate in rapid fading as opposed to slow fading?

   (Short answers to the six questions above are expected, at most a couple of sentences.)

2. Consider N-branch diversity with selection combining. The diversity branches are statistically independent and on average equally strong. The cumulative distribution function of the signal to noise ratio (absolute value) after combining is

$$F(\gamma) = \left(1 - \exp(-\gamma/\bar{\gamma})\right)^N ,$$

   where $\bar{\gamma}$ is the average received SNR of each diversity branch, γ is the post-combining instantaneous received SNR. A user is enjoying a service that requires instantaneous SNR = 5 dB for proper reception, at least 95% of the time. What is the average required SNR in case there is no diversity and in case there is two-branch diversity?

3. Consider a system where information is sent by a signal either being present or not (on-off keying). For signal detection we use a power detector. If the signal is off the detector measures only the instantaneous noise power n and when the signal is on the detector observes signal plus noise power $s+n$. The instantaneous noise power has exponential distribution $p_1(n) = \frac{1}{P_N}\exp\left(-\frac{n}{P_N}\right)$, where P_N is the average noise power. The signal plus noise power also has exponential distribution

$$p_2(s+n) = \frac{1}{P_S + P_N}\exp\left(-\frac{s+n}{P_S+P_N}\right)$$

   where P_S is the average signal power. At the optimum decision level u, the PDFs of the noise power and the signal-plus-noise power are equal, $p_1(u) = p_2(u)$. Determine the optimal decision level. Can it be expressed in terms of $SNR = \frac{P_S}{P_N}$ only?

   Give the function describing the average decision error.

$\gamma = s_1 + n$

$P(\gamma - s_1 \overset{u}{)} = P(\gamma - s_2 \overset{u}{)}$ $\quad s_2 = \sqrt{P_s}$

$\frac{1}{P_N} e^{-u/P_N} =$

4. Consider a channel with two multipath components. The inter-arrival time of the paths is very small compared to the symbol length. The channel can be modelled as

$$h = a_1 e^{j\phi_1} + a_2 e^{j\phi_2} = 1 \cdot e^{j\phi_1} + 0.5 \cdot e^{j\phi_2}$$

The noise power spectral density is $N_0 = 0.05$.

a) Compute the signal-to-noise ratio (SNR) if the components are summed together coherently in phase (constructive interference).

b) Compute the SNR if the components are combined destructively (the phase difference between them is π).

c) Assume that half of the time the receiver sees the constructive and half of the time the destructive combination of the components. BPSK signaling is used. What is the average bit error ratio?
In your computation you can approximate the Q-function as $Q(x) \sim e^{-x^2}$.

5. The receiver receives a signal over two independent channels. The received amplitudes are $a = [6\ 2]$. In both channels the noise variance is the same, $\sigma_N^2 = 4$.
a) Calculate the received SNR if the receiver uses only the first channel. 9 , 1
b) Calculate the received SNR for maximum ratio combining (MRC). 9+1 = 10
c) Calculate the received SNR for equal gain combining (EGC). 5
d) Express the gain achieved while using MRC and EGC compared to using only the first channel 1as in a).

$y_1 = h_1 x + n_1$

$y_2 = h_2 x + n_2$

$\frac{E((y_1 - n_1)^2)}{E(n_1^2)}$

$\frac{MRC}{} = \frac{10}{9}$

$\frac{E\{y_1^2\} + E\{n_1^2\} - 2E\{y_1 y\} E\{n_1\}}{E\{n_1^2\}}$ one

$\frac{EGC}{one} = \frac{5}{9}$

$(y_i - n_i)(y_i^* - n_i^*)$

$|y_1|^2 + |n_i|^2 -$

$y = \sum_{i=1}^{2} y_i a_i e^{-j\phi_i} = (a_1 e^{+j\phi_1} x + n_1) a_1 e^{-j\phi_1} + (a_2 e^{j\phi_2} x + n) a_2 e^{-j\phi_2}$

$(a_2^2 + a_1^2) x + n_1 a_1 e^{-j\phi_1} + n_2 a_2 e^{-j\phi_2}$