

AALTO UNIVERSITY SCHOOL OF SCIENCE AND TECHNOLOGY

Exam January 13th, 2010

Department of Signal Processing and Acoustics

S-88.3105 Digital Signal Processing Systems (5 cr)

Write in each answer paper your name, department, student number, the course name and code, and the date. Number each paper you submit and denote the total no. of pages. 4 problems, 28 points total. Exam problems in English only. Please feel free to answer in Finnish or English. No additional material is allowed in the exam.

1. (8p) Explain *briefly* the following concepts:

- (a) Nyquist filter
- (b) Uniform DFT filter bank
- (c) Perfect reconstruction property in quadrature-mirror filter banks
- (d) Commutator

2. (6p) Prove that for a linear-phase filter whose impulse response has length ML , its M polyphase components should satisfy

$$E_j(z) = \pm z^{-(L-1)} E_{M-1-j}(z^{-1})$$

3. (6p) Let $x(n)$ be an arbitrary sequence and let $x_1(n)$ be the difference sequence

$$x_1(n) = x(n) - x(n-1)$$

and $x_2(n)$ the sum sequence

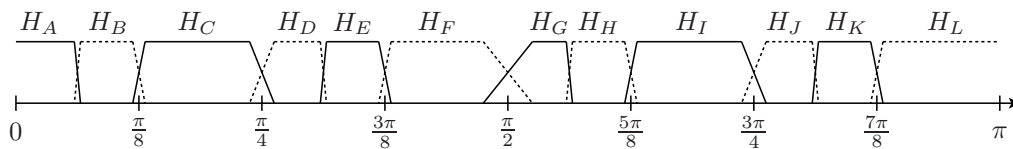
$$x_2(n) = x(n) + x(n-1)$$

Consider two sequences $y_1(n)$ and $y_2(n)$ defined by

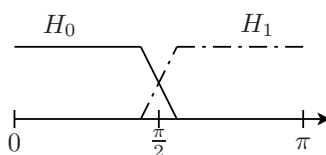
$$y_1(n) = x_1(2n), \text{ and } y_2(n) = x_2(2n)$$

Can we recover $x(n)$ from $y_1(n)$ and $y_2(n)$?

4. (8p) Consider a 12-channel single-level analysis filter bank with unequal passband widths that are shown below.



- Assuming the parent lowpass and highpass filters $H_0(z)$ and $H_1(z)$ with the spectra shown below, sketch $H_0(z^2)$, $H_0(z^4)$, $H_0(z^8)$, $H_1(z^2)$, $H_1(z^4)$, and $H_1(z^8)$.



- Draw the block diagram of the equivalent four-level analysis filter bank. The filter bank should exploit only down-sampling by two, and the parent filters $H_0(z)$, $H_1(z)$.
- Express the filters $H_A(z)$, $H_B(z)$, \dots , $H_L(z)$ in terms of the parent filters $H_0(z)$ and $H_1(z)$.